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Rethinking Learning

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The term "learning" has always covered a wide range of phenomena-from schoolchildren learning their multiplication tables to deans learning that their budgets have been cut. Suddenly, In just the last few years, the range has gotten much wider-by orders or magnitude, it seems. We now have learning corporations, even learning societies, institutional memory. and distributed expertise. Marshall McLuhan's catchy phrase, "learning a living," has been resurrected as a characterization of what life will be like in the fiber optics age.

Traditional conceptions of learning do not comfortably embrace these newer notions. The fact that people are able to talk about them Is mainly a testimony to the remarkable flexibility of language. The term "learning'* has been extended metaphorically, but it does not follow that our basic understanding of learning has changed.

The conception of learning that has guided education through recent millennia is grounded in what has come to be called "folk psychology" (Bruner. 1990; Stich, 1983). To say this Is no particular aspersion on educators, for the same could be said about all the other professions and disciplines In which considerations of human mental activity are Involved-law. psychiatry, theology, even contemporary work in artificial Intelligence.

A number of new views of learning have begun to have an impact on education. "Constructivist," "situated," and "sociocultural" arc labels much in evidence. These labels do. in fact. point to new possibilities. often, however. the new ideas are grafted on to the unexamined assumptions of folk psychology. thus losing much of their innovative potential. Our objective In this chapter is to examine the basic assumptions about learning that come to us as part of folk psychology and to propose an expanded set or concepts that we believe are necessary in order to move educational thought and practice significantly beyond where they are today.

Folk Theory of Mind: Mind as Container

Folk psychology is simply the psychology a person acquires through growing up in a human society. The psychology acquired growing up In a modern western society posits, for each individual, a mind. This mind contains things such as beliefs, desires, Intentions, and memories of past events. These mental contents determine behavior, In the strong sense that if you knew the contents of someone else's mind you would be able to predict their behavior.

The idea of the mind as an Immaterial entity, distinct from the brain, has been bothersome to some scientists. There have been recurrent efforts to eliminate it as a scientific concept. The most notable effort was behaviorism, but recent advances in neuroscience have led to renewed proposals for elimination (e.g., Churchland, 1986). Whether such proposals have scientific merit is an issue that lies outside this chapter. Our concern is with developing a conceptual framework that is adequate to advance educational thought. We can perfectly well admit that the mind is a fiction. The question is whether It Is a useful fiction. Do we really need it? Is it useful throughout or only for certain purposes? Does It lead to pitfalls that we must be on the watch for?

Such otherwise dissimilar thinkers as Jerome Bruner (1990) and Zenon Pylyshyn (1984) agree In arguing for the necessity of a mentalistic level of description. Without It, they show, the accounts we can give of human behavior are simply too impoverished to permit us to make sense of it. We find their arguments compelling. On the assumption that most readers will be of similar persuasion, we shall not review those arguments here. Instead, we shall take It as given that education needs a concept of mind. The question we shall pursue from that starting point is whether education needs or must be limited to the concept of mind that comes to us from folk psychology.

In folk psychology, the root metaphor is mind as container (Lakoff and Johnson, 1980). Beliefs and desires have been the Items of mental content most interesting to those studying the development of folk theory of mind In children (Astington, Harris, and Olson, 1988). But folk psychology places no particular restrictions on the kinds of things the mind may contain. When Freud proposes unconscious motives or Piaget proposes structures d'ensemble or Newell and Simon propose production systems, folk psychology has no trouble admitting any of these as objects that may populate the mind. Some objects have only a fleeting presence In the mind, whereas others reside there more or less permanently. Learning is any process by which these more enduring objects get into the mind. Again, folk psychology is quite permissive regarding the processes by which new objects get into the mind. It can tolerate such shortcut notions as knowledge transmissionknowledge passing from the mind of the teacher to the mind of the student via the spoken word. But it can equally well accommodate the constructivist notion that knowledge is produced within the mind through mental activities of the learner. And there is no fundamental objection to the Vygotskian idea of internalization, although It is likely to appear obscure. As for the so-called "cognitive revolution," its main effect for educators and related kinds or practitioners has been to reinstate folk psychology from its exile by behaviorists (Carroll, 1976).

Given all this versatility, one might suppose that folk psychology would be safe from complaints that It stands In the way of grasping and applying new ideas. Yet that is the complaint we shall make. It may be true that with sufficient effort one can think anything that needs thinking, without violating the premises of folk psychology. But folk psychology does have Its biases. Some ways or thinking are much easier to pursue than others, some ways seem more natural. Generally, It appears that people are most comfortable dealing with cognitive issues when they are formulated in ways that fit with the mind-as-container metaphor. Hence:

• Knowledge is most readily conceived of as specifiable objects In the mind, such as discrete facts, beliefs, Ideas, or Intentions. Less specifiable sorts of knowledge, such as Intuitions or understandings, are harder to hang on to, and so are any sorts of knowledge not assignable to Individual minds, such as the "state of knowledge" in a discipline.

• Mental abilities are defined in terms of doing specifiable things with specifiable mental objects. Memory involves retaining and retrieving such objects. Classification and sequencing abilities (favorite components of thinking skills programs) involve arranging mental objects, and reasoning abilities involve new mental objects (conclusions) resulting from combinations of existing objects. Mental abilities that cannot be thus specified tend to be neglected, treated as mysterious, or reduced to specifiable operations on specifiable objects.

There are important kinds of learning that do not readily fit the container metaphor. They are difficult to teach by ordinary methods, because pedagogy has evolved as a craft for cultivating mental content. They are difficult to test, because educational testing of course depends heavily on the container metaphor; it is a matter of inventorying mental contents. Because they are unspecifiable in mental content terms. they may also be difficult to defend as educational objectives. At the same time, there are important kinds of mislearning or problematic learning that ought to be taken into account in education but that are difficult to come to grips with unless they are reduced to mental content. Finally, there is the whole matter of the advancement of knowledge. it often takes the form of specifiable objects-theories, Inventions, discoveries, and the like-but these tend to take on a life of their own that is difficult to reconcile with these same objects being considered as content Inside individual minds.

In the following sections we shall elaborate on these kinds of learning that do not fit the container metaphor. They prove to cover much more educationally important territory than we might at first imagine. At the same time we shall try to show how concepts drawn from connectionist approaches to artificial intelligence. sociocultural theory, and philosophy of science provide ways out of the mind-as-container trap, enabling us to better comprehend human learning in its full range.

Mind Without Mental Content

There are kinds of learning that cannot be reduced to specifiable objects or actions. There is a large class of learnings that we may describe as "learning one's way around in" some physical or abstract domain. The salient example is place learnIng (Woodworth, 1958), which means knowing some physical environment well enough that one Is never lost in it and can quickly find one's way from one place to another by an efficient route. Experienced taxi drivers have it, and so do rats or mice In a building. Such learning is sometimes spoken of as acquiring a "mental map" (thereby identifying it with an object In the mental container). But mental maps would seem to characterize an early stage of place learning. At high levels of place learning, one does not have to consult a map, mental or otherwise.

Closely analogous to place learning, and more immediately relevant to educational concerns, is number sense. Number sense is "knowing one's way around" numerical domains. It manifests Itself In the ability to make flexible. efficient, and almost effortless use of mathematical facts-for example, that 63 is closer to 100 than it Is to zero but closer to 50 than to 100, that it is odd. that It Is divisible by 7 and 9, that adding 7 to It makes 70-as they are encountered In situations that call for their arithmetic manipulation. It is the basis for shortcuts, error checking, approximation, and taking advantage of situation characteristics. Scribner (1984), for instance. found dairy truck loaders taking advantage of the row-and-column layout of packing cases and basing their calculations on those. Any particular number facts and operations can be accounted for by the relatively small number of principles constituting arithmetic, and these might plausibly be regarded as items of mental content. But to account in this way for the flexibility and situatedness of normal quantitative skills, we would have to posit an inordinate number of little context-sensitive rules. And even then we would have trouble accounting for how number sense works in new situations.

"Creativity" is a name applied to all kinds of productive thinking that cannot be explained by rules and Items of Information In the mind (Bereiter and Scardamalia, 1993, chap. 5). Hence the perennial belief in Inspiration, attributing creativity to a source (possibly divine) outside the mind. But human Intelligence exhibits creativity almost continuously, In countless small ways that hardly justify divine intervention. In conversation we frequently start a sentence without knowing how we will finish it. Sometimes we get Into trouble and have to start over, but more often we improvise effortlessly, producing a sentence that is both grammatical and faithful to our intent. Such mundane creativity, which represents the typical rather than the exceptional workings of the human mind, similarly defies explanation in terms of specifiable mental content.

Another major kind of learning not suited to the container metaphor is that occurring in the study of literature. People who appreciate literature themselves seldom question its value in the curriculum, but they have trouble defending it during back-to-basics crazes. They can claim that literature enriches life, makes one a better person. changes the way one sees the world-but what, precisely, is learned that could give substance to these claims? The dilemma shows up most clearly when it comes to testing. Students can be tested on their knowledge of text content or facts about the author and the historical context, or they can be asked to interpret passages or write critical essays-but what is the learning that is supposed to be reflected In such tasks? The teaching of literature tends similarly to vacillate between teaching specific but Incidental information and becoming an occasion for students to talk about whatever reading has brought to mind.

The container metaphor is thus severely limited in its applicability to human learning and Intelligence. But what alternative is there? As recently as 1985, Jerry Fodor could say or cognitive theory based on the container metaphor, that it is "the only game in town" (p. 90). A year later, however, a monumental twovolume work firmly established a second game, now generally known as connectionism (McClelland, Rumelhart, and PDP Research Group, 1986; Rumelhart, McClelland, and PDP Research Group, 1986).

As with folk psychology, the scientific status of connectionism Is not of immediate concern. We may look on It as simply providing a new metaphor for the mind. But connectionist achievements In artificial intelligence (Al) are important in making the metaphor plausible-indeed, in making the metaphor even comprehensible. Traditional A[is fully consistent with the mind-as-container metaphor. The builder of an Al implementation can tell you exactly what rules and Items of Information are contained in the simulated mind. But a connectionist Al program does not contain any rules or facts. A connectionist program consists of a set of abstract objects that Interact with each other and with the outside world according to purely quantitative schemes. It is essentially a pattern detection device. Give It patterned Input, such as spellings of English words (irregular as well as regular) or different portraits of the same people or Information on animal characteristics and it will begin to act as If It is making progress in figuring out the underlying patterns, categories, defining rules. and so on. But such pieces of knowledge are nowhere to be found. As Rumelhart (1989, p. 135) put it, "all the knowledge is in the connections" - and these connections are merely quantitative.

The connectionist mind can thus be knowledgeable without containing knowledge (Bereiter, 1991). It will act In ways that we normally attribute to mental content, as we do with human beings, but in this case we know the content Is not really there. More importantly for our purposes, to the connectionist mind, place learning, number sense. and linguistic creativity are all perfectly normal capabilities that It may acquire, and there is nothing mysterious about claiming that literature alters the way we see the world. It is exactly what we expect of a mind that attunes itself to pattern in the information It receives.

The new metaphor suggested by connectionism is mind as pattern recognizer. There have been important proposals along this line that preceded the rise of connectionism (Lakoff, 1987; Margolis, 1987; Shaw and Turvey, 1981), but it required connectionism to provide a conception or how such a mind could actually work. For the idea is not that the mind stores up patterns and matches new experiences to them (that is how folk psychology would conceive of it). The idea is that the mind acquires abilities and dispositions to recognize and respond in various ways to various patterns, but the patterns are not in the mind. We can say that the patterns are In the environment or, more cautiously, that the patterns are a way for us as observers to describe relations between the mind and the environment.

None of this seems as easy or natural as defining learning in terms of mental content, but until educators can begin to think about learning in this way, we believe, many of the most important kinds of human learning will remain In the shadows.

Knowledge Embedded In Practice

In Rome, to cross a wide thoroughfare where automobile traffic flows endlessly, pedestrians will step out boldly and proceed at a steady pace while the automobile traffic parts like the Red Sea around them. Trying that in a North American city would create a horrendous traffic jam, if not much worse. To attribute the difference to the superior skills and versatility of individual Roman motorists is hardly adequate. What we see In Rome Is an evolved motoring practice that adapts to pedestrian cross-traffic. No amount of Individual training of motorists would produce it. There is, Indeed, probably no way to get such a motoring practice established in places where it does not already exist. It would have to have started long ago, when cars were few and not very fast, and become progressively refined as traffic grew denser and raster. Thus we are looking at a learning trajectory that extends over several generations and for which the necessary Initial conditions no longer exist. Today's Roman motorists are collectively executing a skilled performance that is more than a combination of their individual driving skills. They are participants In a social practice with a learning history of its own, growing out of the activities of drivers now gone, who probably had no Idea what they were contributing to.

Some cultural practices, such as school teaching and social drinking, have longer learning histories. Others run their course very quickly, as when flood or earthquake victims create a temporary society In emergency accommodations. In all cases, however, we witness learning that cannot be fully accounted for at the level of individual minds, connectionist or otherwise. The cultural practice itself is a kind of knowledge and the evolution of that practice is a kind of learning.

The idea of knowledge and learning inhering in cultural practice now enjoys considerable currency, as an aspect of situated cognition. Accordingly, we will not elaborate on it to the extent that we did on connectionism. Several other chapters in this book develop the idea more fully and In various contexts (see Case, this volume, chap. 5: Rogoff, Matusov, and White. this volume, chap. 18). Instead, we shall focus on the conceptual requirements for dealing effectively with this idea in education.

Collaboration and teamwork are very much In the air these days, and have been ever since management experts began attributing much of the success of Japanese Industry to teamwork and group loyalty. "Ability to work in groups" is now appearing as an educational objective almost on a par with literacy and numeracy. How do we address or think about such an objective?

Situated learning has caught on so easily and quickly among educators that it seems safe to assume that most have found a way to assimilate It within folk psychology. Knowledge and learning beyond the individual level can be treated as metaphorical extensions of "real" learning-as occupying a larger container, so to speak. And the skills embodied in cultural practices can be treated as individual social skills, which are skills like any others, except that they are learned through social experiences.

There is nothing dramatically wrong with these folk versions, but they do reduce the likelihood of any very profound change taking place in educational practice. Thomas Rohlen (1989) has studied Japanese socialization practices from kindergarten to the workplace. The story he tells is not simply one of children acquiring social skills at school, which they carry with them into adult life. Rather, schools foster the development of certain group practices, related to Internal maintenance of discipline and performance standards, which are replicated in the adult workplace. Thus an Important part of what passes from school into society is the practices themselves, not just the individual knowledge that goes with them.

Social practices evolve in all schools, of course. Some of them, such as recitation, are so salient that children often start school already knowing them, having learned them through playing school at home. But such practices have little transfer value beyond school. Unfortunately, the social practices that seem most likely to have transfer value outside school are of the more subversive sort that develop without official sanction. It has been observed, for instance, that students evolve subtle and coordinated ways of lowering the level of teachers' work demands. This Is clearly more than a collection of individual skills. Individual students may not even be aware of the point of the collective activity they are engaged in (that is not uncommon in social practices). Yet the practice itself may well be transferable to other situations, with other participants, where it may serve similar purposes of workload reduction.

What if schools could foster the development of cultural practices that have broad applicability to situations in the outside world that call for working productively with knowledge? It sounds like just what the "Education for the Twenty-first Century" and the "Global Competitiveness" pundits keep urging upon us, except that they never quite put it that way. Instead they talk about higher-level thinking skills and group participation skills. Later we shall argue for -knowledge building" as such a transferable cultural practice. But in order to grasp the general concept, we have to recognize that promoting individual learning (whether it is done individually or through group processes) Is one thing and fostering knowledge embedded in transferable social practices is another. These represent two jobs for schools, not one, even though they are closely intertwined. The second job receives virtually no attention in educational thought and practice, because our folk conceptual framework offers essentially no support for doing so.

Knowledge as Object

Although folk psychology treats knowledge as objects In individual minds, there is another way of treating knowledge. which has a long and respectable past. In the history of knowledge (which includes the history of ideas and the history of science), knowledge has been treated implicitly as consisting of objects that exist in some sense independently of individual knowers. It matters, of course, that various people believe or disbelieve, understand or misunderstand an idea. That is much of what the history of ideas is about. But essential to such history is the idea itself, the object of people's attitudes and actions. Thus phlogiston, an idea, plays a role in the history or chemistry In much the same way that Charles 11, for instance, plays a role in the political history of England.

For the most part, historians as well as less specialized thinkers about knowledge have been little bothered by the problem of reconciling the two views-one a subjective view, regarding knowledge as a mental state, the other an objective view, regarding knowledge as something In its own right. Educators have lived comfortably with this dualism. One moment the teacher wears the hat of a biologist or space scientist, and knowledge is treated as something brought before the class to be discussed and understood. The next moment the teacher puts on a pedagogical hat and asks questions to probe knowledge, now regarded as stuff hidden away in the minds of individual students.

This casual dualism obviously works, up to a point. We believe, however, that education is now at a point where It needs a more disciplined approach to knowledge. We believe, as everyone is saying, that education will have a crucial role in determining how successful societies are in negotiating a shift from manufacturingbased to knowledge-based economies (Drucker, 1994). Successful societies will be those whose citizens are most adept at creating and elaborating the uses of new knowledge. In industrial societies, there is not much problem distinguishing knowledge from what it is applied to. If you are educating aeronautical engineers, say, then the knowledge you are concerned with is knowledge of the kind conventionally located in the mind of the engineer. The objects to which the knowledge is applied are huge metal contraptions that would never be confused with things inside people's heads. But If you are educating a knowledge worker, then you again have knowledge conventionally located In the mind of the worker, but what that knowledge is applied to is also knowledge. Now there is considerable risk of confusion between workers' knowledge and the objects to which it Is applied.

If educators are going to play more than subservient roles in a knowledge-based society, they are going to have to be able to negotiate flexibly and without confusion between several different ways of conceptualizing knowledge, appropriate to its different roles. At present, this is a bit like negotiating a complex business arrangement between four companies that are all named General Motors. We do not have distinctive terms for various senses of the word "knowledge," and the term "learning" is applied indiscriminately wherever a knowledge- related verb is required.

Philosophers of knowledge. although living In A less complicated world than modern educators, have had to wrestle with related conceptual problems. Perhaps the most fully developed effort to get knowledge problems sorted out was that of Sir Karl Popper, building on work of Gottlob Frege. Popper (19 72) identified three separate though related worlds: World 1, the physical world; World 2, the subjective world, the world inside our minds; and World 3, the world of what he called, to the confusion of many. "objective knowledge." What he meant was not "objectively true" or "free of subjective bias." He meant, simply, *treatable as an object*. Thus phlogiston theory clearly belongs in World 3, although no one any longer would claim It to be true. Indeed, It has no possible home other than World 3. It does not belong in World 1, because according to present knowledge no such substance as phlogiston exists In the natural world. And, assuming that no living person believes in phlogiston theory, It cannot be said to exist in World 2 either. But as an object In World 3, It survives as a historically Important theory. It can be studied, its history can be told, people can judge its effects on the progress of chemistry, analyze its appeal, and so on.

As with connectionism and knowledge embedded in practice, we do not put Popper's three-worlds schema forth as something forced upon us because of its truth or universal desirability. We put it forth as having something to add to a conceptual framework that education needs in order to advance. However, in this case the addition to the conceptual framework is mainly of value In helping education to develop better connections with the outside world. In the past this connection has been a simple one. The world outside the classroom is the source of knowledge (this outside world, or course, includes the sciences and scholarly disciplines: It Is not just the world of practical affairs). The teacher's job is to get some of this knowledge into the heads of students. Some small number of students may eventually become producers of knowledge themselves, a possibility that begins to receive attention at the university level, but this is seldom a concern of the schools. In Popperian terms, the traditional job of the schools Is to select objects from World 3 and get them transformed Into objects in World 2, the minds of Individual students. But if that is all there is to it, we hardly need Popper's framework. rhe ordinary language of curriculum planning should suffice.

The Implication of the term "knowledge-based economy," however, is that wealth generation throughout the economy depends on creating and transforming knowledge (Romer, 1990). (Using knowledge does not distinguish a knowledge-based economy; all economies, including the most primitive and stagnant. rely on vast amounts of knowledge.) It would seem that this rise in prominence of knowledge should have some educational implication, even if it is not clear what. Perhaps the least controversial implication is that educators ought to be thinking seriously about what the implications are. They should not be jumping directly to conclusions-that more knowledge should be taught, or less; that computer and media literacy are the answer, or lifelong learning dispositions; that it all comes down to information search skills. or, alternatively, promoting curiosity and creativity or cooperative learning skills; that we must teach knowledgeprocessing skills. (Fine. but what are they? Are they just yesterday's thinking skills with the dust blown off?) These are all Ideas (possibly good ones) arising from word associations. Where is the kind of thinking that arises from deep analysis? We suggest very little of It Is going on, and that a major reason Is the lack of conceptual tools for doing the analysis. Popper's three-worlds schema is not exactly a precision Instrument, but compared to what folk theory has to provide, it gives a significant boost to analytic powers, as we shall try to show In later sections.

An Expanded Set of Concepts for an Expanded Approach to Formal Education

Ideally, education should have a comprehensive theory of mind and knowledge that can deal in a coherent way with the various ideas touched on in preceding sections. Lacking that, however, we can at least assemble a group of concepts that in combination allow necessary distinctions to be made and that support the development of new ideas. We believe the essential concepts can be assembled out of Popper's three-worlds schema, ideas coming from connectionism and situated cognition, and other ideas retained from folk psychology. This may strike some as naively eclectic;

for there are situated cognition proponents who sneer at the mention of Popper, upholders of folk psychology who regard connectionism as a throwback to behaviorism, and connectionists who think the Idea of mind Is obsolete. But there are fundamentalists everywhere, and we need not be swayed by their exclusionary passions. The fact is that we need a patchwork of concepts, because no single approach can handle all the ways In which knowledge needs to be considered by modern educators. There is no reason to get spiritually agitated about this. Most people who work In applied sciences, whether they deal with water quality, highway safety, or telecommunications, find themselves in a similar situation.

Although it does so in a rather heavy-handed manner, Popper's three-worlds schema lays down what seem to us fundamental distinctions necessary for clear thinking about educational processes. An exemplary classroom discourse is likely to be about World 3 objects-ideas, theories, interpretations. and the like-but these objects *refer* to World 1-experiments, observations, remembered experiences, and so on. However, the classroom discourse is orchestrated by the teacher *for (lie purpose of* producing changes in World 2, the mental states of the students. Leave out World 3 and you either have naive realism, in which students' beliefs (World 2) are to be brought into conformity with the true nature of things (World 1), or else you have the relativistic gabfest. in which students "share" thoughts out of their respective Worlds 2. with no basis for comparing or improving them. Leave World I out of consideration and you have the verbalism academic discourse is prone to. Leave World 2 out or consideration and you leave out personal meaning and the intuitive wellsprings of progress in World 3.

Popper's schema must be augmented, however, by what could be called World 2.5. It Is the world of knowledge embedded In practice. As we have already seen, this is knowledge that cannot be reduced to World 2, individual minds. But it is not part or World 3, either. Being embedded in practice means, in fact, that knowledge has not been abstracted as objects that can be discussed, compared, hypothetically modified, and so on. In addition to recognizing knowledge embedded in practice, situated cognition theorists also recognize knowledge embedded in tools (such as measuring devices) and artifacts (including books). These may be physical embodiments of World 3 knowledge, but they are not World 3 knowledge Itself. If students are discussing a theoretical article, they are of course not discussIng the pieces or paper it is printed on; but generally they are not discussing the text or the article, either. They are discussing the theory presented in the text. That is the World 3 object, and it is Important that it be distinguishable from its wrappings. It Is the theory itself, not the text through which it is presented, that students will argue is true, false, interesting, trivial, applicable or inapplicable to the problems that concern them.

The importance of the World 3 concept becomes most evident when we consider knowledge in relation to goals of education. Correspondingly, the difference between connectionist and folk conceptions of mind takes on special importance when considered in relation to World 3. If we take the folk psychological view of mind, then World 2 Is a container of representations of World 1 and World 3: education is concerned with the adequacy of those representations; but the distinction between World I and World 3 can for the most part be ignored. We may, for Instance, want students to know where Egypt is, but it is rather a pedantic issue whether this means knowing where Egypt Is physically situated on Earth (World 1) or whether It means having It properly located on a world map (the result of a World 3 construction).

If we take a connectionist view of mind, however, then the relation of the student's World 2 to Worlds I and 3 becomes more complex and the difference between I and 3 takes on more significance. The student's mind is seen as adapting to patterns, both as these are experienced in the physical world and in social practices. That kind of adaptation to pattern, however, characterizes the learning of social animals as well as human beings. It Is what we described earlier as "learning one's way around" In physical and social environments. A distinctively human aspect of cognition is learning related to World 3. Educated people learn their way around In this world, as well. That is, they learn their way around in a world of ideas, explanations, problems and issues, theories, stories, histories, critiques, and models of various kinds. To reduce this, as folk psychology does, to acquiring mental copies of these objects, is to omit most of what is important in acquiring such knowledge. It is knowing how to function effectively in this world that counts, appreciating what is to be found and experienced there, feeling at home in a World 3 that is continually expanding as one explores more of it.

There is another distinctly human aspect to cognition, which folk psychology tends also to neglect, and in which it is important to recognize the several different worlds that individual minds can relate to. This is the creative, constructive capacity of human cognition. We do not just learn our way about in the physical world as it is, we modify that world, sometimes drastically, to our purposes. We devise new social arrangements and practices as well. We do not necessarily take World 3 as given, either. We keep adding new objects to it and modifying or reevaluating old ones. In the context of creative activity. World 3 has a special significance. Some changes that we bring about In the physical and social worlds evolve gradually and unconsciously, but those that are the products of deliberate thought and imagination are often worked out first In World 3. They originate as Ideas, plans, or designs, which are discussed and modified In their abstract form before they are enacted concretely. What it means to have a knowledge-based society is that a very large part of productive activity goes on In World 3 In comparison to the amount that is directly engaged with physical things and overt practices.

Although schooling Is extremely limited In Its means for supporting students' activities in the physical world and for enabling them to participate in real-life social practices, It Is well designed to support activity in World 3. That, of course, has been Its traditional role-to transmit the knowledge contained In books. Dissatisfied with that limited role, educators throughout this century have strived to bring more of the physical and social worlds into schooling. Although these efforts have not been without effect, schooling continues to provide only a severely cramped and often distorted range of experience with natural and social phenomena. How much more promising would It be for educators to try Instead to provide students a richer and more rewarding involvement in World 3!

Advocates of liberal education have always claimed much more for formal education than merely stocking the mental warehouse. The benefits they have claimed for immersion in the world of books and Ideas have included wisdom, imagination, and character (e.g., Barzun. 1944). But these larger benefits have been shadowy ones, easily lost sight of. The conception of learning provided by folk psychology provides no realistic way of talking about how students might get wisdom, Imagination, or character out of a book. They might learn specific lines of verse, or specific ideas and arguments, and conceivably these might be retrieved on important occasions and used to some purpose. But we know that is unlikely and that it furthermore misses the point.

From a connectionist standpoint, the retention In memory of particular items of text or information is incidental and a challenge to explain. The natural and expected result of sustained exposure to a culture's classics would be very general kinds of pattern learning, which would be manifested In ways such as the following:

1. A familiarity with, a feeling of being at home in and in harmony with the attitudes, objects of interest, and ways of thinking pervading the classical body of work.
2. A change in the way we see the world, through what amounts, in Lakoff and Johnson's (1980) terms, to a change in the "metaphors we live by."
3. A change in our intuitive standards of moral and aesthetic judgment. In short, the primary educational results would be in such attributes as wisdom, imagination, and character.

At one level of educational planning these more profound outcomes of schooling are taken very seriously. They are much at issue In the controversies that have been swirling around university English departments and public school boards concerning multiculturalist movements to revise the literary canon. The disputes are over what culture students are to feel at home in, what ways of seeing the world they are to internalize, whose moral and aesthetic judgments are to serve as standards. But at the level of pedagogy-the level of classroom lessons, projects, and tests-all this tends to be forgotten. And that Is to a considerable extent, we believe, due to the conceptual Impoverishment of the folk psychological view of learning.

If the mind-as-container metaphor fails to do justice to the deeper outcomes of reading and discussing the received works of World 3, It falls even more conspicuously to do justice to education's potential role In developing people who are active constructors of and workers within World 3. We will elaborate on that issue In the next section. To round out the present discussion, however, we need to recognize the continuing value of certain aspects of folk psychology as part of a conceptual framework for educational thought.

There are two ideas grounded in folk psychology that seem Indispensable to a modern pedagogy. One idea is probably as old as folk psychology itself The other has only been fully developed within the past two decades. The old idea is intentionality. Defenders and critics of folk psychology agree on its centrality. Folk psychology makes sense of and predicts people's behavior on the basis of inferred intentions. It is hard to imagine a teacher carrying on any reasonable schooling process without attention to the students' own goals, and folk psychology provides the only practical basis for doing so. The newer idea is mental models (Gentner and Stevens, 1983; Johnson-Laird, 1983). A mental model is a more systematic version of the traditional idea of *belief*. Mental models are especially applicable in cases where students' behavior appears to have some logic to it, but not of the conventional or intended kind-when they commit nonrandom errors in arithmetic, for Instance, or ask strange questions. such as "What keeps gravity from leaving the earth?" In these cases, a useful strategy for the teacher Is to Imagine a theory or rule that might be entertained by students and that would give rise to the observed behavior. Folk psychology and its modern variants encourage teachers to believe that they are making inferences about mental models that are really there in the minds of students, although not directly observable. From a connectionist viewpoint, such conjectured models serve only a heuristic purpose for the teacher, but they can be quite useful in that role.

In this section, we have not introduced any new educational vision. We have merely taken educational ideas variously advanced by traditionalists, constructivists, socioculturalists, and just plain folks, and tried to show how an expanded set of concepts of knowledge and learning Is necessary In order to do justice to them. The existing conceptual framework leads to a dreadful discontinuity between big ideas and day-to-day pedagogy. At a high verbal level, we can spin out grand ideas of what education might be, but at the classroom level we are bound to a psychology far older than any of the big ideas. a psychology ill-suited for coming to grips with what It means to understand or to invent. In later sections, we shall try to show how an expanded set of concepts better enables us to tackle such higher concerns, and how this In turn can begin to generate new educational vision.

Understanding Newton's Dog: A Connectionist View of Understanding

"Learning with understanding" is one of the catch phrases of educational reform at present. What does it mean? With the kind of partly systematized folk theory usually applied to such a question, quite different accounts are given, depending on whether we are talking about what it means to understand Newton's theory, for instance, or about what it means to understand Newton's dog. A more enlightened theory of mind, we argue, will eliminate this discrepancy.

A common present-day account of understanding Newton's theory would run something like this: Through everyday experience, the student has already acquired a mental model of the physical world and how objects move in it. On studying Newtonian mechanics, any of several things may happen. The student may acquire a new cognitive structure suitable for answering test questions and solving textbook problems related to Newton's laws, while the student's original mental model remains unchanged. This will be revealed by presenting novel tasks, such as describing the path of an object dropped from an airplane, Alternatively, the student's mental model may undergo changes so that it becomes more like the model that was in the mind of Isaac Newton (or In the mind or a certified physicist, acting as surrogate). Teachers are now being encouraged to look for such shifts by having the students draw actual maps of their conceptual structures (networks of concepts and relationships), which may then be compared to maps prepared by experts (cf. Novak and Gowan, 1984). What it means to understand, accordingly, is to have the thing that is in your mind correspond to what is correct or to what is in the minds of people who are already assumed to understand. This is folk theory of mind, through and through, no matter how it may be dressed up with contemporary cognitive science terminology.

Understanding Newton's dog, however, is viewed in a more commonsensical way. In the first place, we would all recognize that Newton's dog could be understood in various ways, that there is no privileged way that ought to be expected of everyone. Not everything will pass as understanding, however. In judging whether someone understands Newton's dog, we would look for the following:

1 . The ability to act Intelligently with respect to the dog. Acting stupidly with respect to it would cast doubt on a person's understanding. Again, it is not a matter or there being just one way to act that Is intelligent. but judgments can be made.

2. The ability to explain problematic aspects of the dog. We would not require a lecture on dog neurology, unless it happened that Newton's dog exhibited neurological peculiarities. In short, the ability to explain is relative to what it is about the dog that seems to need explaining.

3. An awareness of the limitations in the two previously mentioned abilities, and a disposition to Improve them. Claiming to know all there is to know about the dog would likely be taken as a sign of limited understanding, as would a disinterest in knowing more.

These commonsense criteria, as we see, make no reference to mental models of dogs or any other sorts of objects in the mind. Understanding inheres in the kinds of things a person is able or inclined to do with respect to the object in question. If the object in question is something like Newton's theory, however, folk theory treats understanding as a characteristic of something In the learner's mind. Understanding no longer inheres in abilities and dispositions; It is instead a matter of correspondence between what is in the learner's mind and external reality or what Is in someone else's mind.

Through a combination of connectionism and Popper's conception of World 3, however, we can approach the understanding of Newton's theory in the same sensible way that we approach the understanding of Newton's dog. Popper's World 3 is a world in which Newton's theory is as real as Newton's dog is In World 1. In the connectionist view of mind, there Is no mental content to talk about. There are only abilities and dispositions. Abilities and dispositions, as we have seen, are all we need to talk about In judging understanding of Newton's dog. If we treat Newton's theory as an equally real thing, albeit an immaterial one, then the same approach may be taken.

Applying the same criteria as before, we would judge a person's understanding of Newton's theory on the basis of (1) how intelligently the person acts with respect to Newton's theory, (2) ability to explain whatever is judged to need explainIng in the theory, and (3) awareness of and interest In doing something about shortcomings on the first two criteria. Of course, similar criteria might be used by a folk cognitivist in assessing a student's understanding. The difference is that the folk cognitivist is using observations as clues for inferring the theory hidden away in the student's mind, which Is to be compared with the kind of theory hidden away In the mind or the physicist; whereas we tire saying that understanding just is these abilities and dispositions. There Is not something else hidden away in the mind that Is our real target, if we but knew how to get at It.

The bottom line of the connectionist view is that if we could open up the mind and probe its depths we would not find anything we could make sense of. The sense is in the dispositions and abilities, of which we or course always have only partial information. We never know for sure what a person will do under other circumstances, or Indeed under the same circumstances at another time. But such uncertainty is built into the cognitive system and its relations to the environment. It Is not a result of the mind's inaccessibility to observation.

An immediate advantage of this approach to understanding is that it permits liberality without a total collapse into relativism. Science educators these days agonize over what to do about students' naive theories vis-a-vis authoritative knowledge. Folk theory of mind ensures that this will remain a muddy problem. If theories are just things in the mind, then who is to say that the thing in the student's mind is inferior to what is in the physicist's mind and should be altered accordingly? If we appeal to truth, we get into one kind of trouble. and if we appeal to elite consensus we get into another: but if we maintain that the student's theory is as good as Newton's, we come off sounding plain ignorant.

Treating Newton's theory like Newton's dog, however, we can approach the matter of instruction in a more forthright way. Newton's theory has won a place in science curricula, whereas Newton's dog has not, because of its importance in a vast range of human concerns. So it is an object students ought to come to understand, in the sense of learning to act intelligently with respect to it and to be able to explain what needs explaining. Acting intelligently with respect to Newton's theory could include being able to recognize it in various guises, to discuss it knowledgeably, to make use of it in various ways, and to recognize what it is and is not good for. Being able to explain its problematic aspects would include, for instance, being able to explain how it can be asserted that a body in motion will continue in motion unless acted upon by a force when all the moving objects we observe eventually come to a halt unless acted upon by a force. The issues of truth and authority, so vexatious to educators of a postmodern persuasion, need scarcely arise. We do not insist that there is some true or correct understanding, but neither do we say that anything goes. Actions and explanations can be criticized, improvements can be urged. All that has happened is that understanding Newton's theory has joined the natural order of human activities, so that it can be treated in the same humane, liberal, and constructive ways that we try to treat all other human activities.

Knowledge Budding Distinguished From Learning

Let us use the term "knowledge building" to refer to the production of knowledge objects-objects that in Popper's scheme occupy World 3. The term is intentionally broad in its denotation, so as to include what scientists and scholars do and at the same time to include similar productive activity on the part of students. Were it not for the inclusion of students, the question of how knowledge building relates to learning would likely not arise. If you ask scientists or scholars at work what they are doing, you do not expect "learning" to be the answer. They will ten you what they are trying to find out. explain, prove or disprove, or what problem they are trying to solve. Of course, they learn things in the process, but that goes without saying.

From time to time, however. the scientist's or scholar's answer to "What are you doing?" would be "learning." Often the response would take the form of "I'm *trying* to learn" such and such-to use a new piece of software, to carry out a difficult experimental procedure, to master a language or a body of mathematics. and so on. Usually the learning would be seen as instrumental to the person's work, the actual work being knowledge production. Thus in the world of knowledge building, learning plays an important, even if occasional role. But it is not perceived as the same thing as knowledge building.

Interestingly. if you ask school students what they are doing. you are not likely question find that students tend to answer by referring to the work to get either a learning answer or a knowledge-building answer. Researchers who have asked the question find that students tend to answer by referring to the work they are doing (Doyle. 1983; Lancy. 19 1976a; Lancy, 1976b). But unlike scientists and scholars, the work, as perceived by students, does not involve the production of knowledge. Rather, it centers around the production of pieces of writing or other artifacts, the completion of assigned problems, and other sorts of schoolwork or self-chosen activity. Learning occurs, of course -that is the point of the activities. But as far as students' goals are concerned, what distinguishes students from practicing scholars and scientists is not that they are focused on learning but rather that they are not focused on either learning or knowledge building.

Neither folk theory nor sociocultural theory makes a distinction between knowledge budding and learning. Popper's three- worlds model provides a basis for such a distinction, although it leaves the nature of learning entirely open. Knowledge building is activity directed toward changes in World 3. Learning is directed toward changes in World 2.

Learning as Complementary to Knowledge Building

The lack of a distinction between knowledge budding and learning has caused needless dilemmas for educational innovators. One innovative group reported with dismay that teachers who were seemingly committed to a constructivist approach to mathematics and were doing fine work in getting students to, in effect, reinvent mathematics, nevertheless started each class session with mental arithmetic drills. It seems to us that these teachers had intuited a distinction that went beyond their adopted constructivist theory.

We have encountered teachers who would never stand in front of a class and explain the difference between restrictive and nonrestrictive clauses and who would be appalled at the thought of teaching young children how to sound out words; yet, when *introducing a* new software application to their students, they do not hesitate to deliver direct lessons, complete with explanations, diagrams. and demonstrations, accompanied by exercises of graduated complexity. Somehow, teaching software use falls outside their strict constructivist philosophies. It is just something the students need to learn in order to get on with their constructivist work.

With a better-elaborated epistemology , such teachers might find that a great deal more of school learning fell into the category of "just something the students need to learn in order to get on with their constructivist work" -their constructivist work being knowledge building. The so-called "basic skill" components of reading, writing, and arithmetic might well fall into this category. So might various kinds of factual knowledge that are presupposed in reading material students will use-knowledge of countries' locations, for instance. and the many items that have been catalogued in the name of "cultural literacy" (Hirsch, 1987). It is not our business to make recommendations about curriculum content here. it is always debatable whether something should be addressed as a learning objective or whether it should be left to be picked up incidentally in the course of knowledge building. Should students be explictly taught principles such as control or variables and the distinction between hypothesis and evidence? The important thing is that teachers should be able to discuss this as a strategic issue and not as a test of loyalty to the constructivist faith.

Practical educators implicitly recognize the kind of distinction we are trying to make here, but, operating within the conceptual framework of folk theory, they tend to express it in misleading and antagonizing terms. They will say, for instance, "I believe in constructivism, but some things still need to be learned by rote." Such a statement Is incoherent. If we believe In constructivism, then we should believe that even rote learning is a constructive process. Implicitly, the position being advanced is one that could be stated more accurately as, "I believe that knowledge building should be the principal activity In school, but some things need to be deliberately pursued as learning objectives." Within the conceptual framework of folk theory of mind, however, such a statement Is uninterpretable. That is why we need a conceptual framework that distinguishes between working in World 2 and working In World 3.

"Constructivist Learning" and Knowledge Building

"Constructivist learning" is a name attached these days to a very large range of classroom activities that do not have very much in common except for the absence of direct instruction. What It means can vary considerably, however. The differences become evident if we compare various approaches according to the extent of actual construction of World 3 objects-that is, Instances of knowledge treated as objects to be considered. criticized, improved. The following are observable varieties that are all typically classified as constructivist:

1. Messing around. Students are provided with equipment or materials and encouraged to do whatever they feel like. A set-up for studying light and shadow, for instance, may end up being used to make shadow puppets (Smith and Neale, 1987). Although learning goes on in such activity, as it does in any deliberate activity, there is likely to be no objectification of knowledge whatever, except possibly by the teacher.

2. Hands-on learning or guided discovery. Here there is guidance In what to do, often with the aim of discovering some particular mathematical or scientific principle. Ideally. a World 3 object-the principle In question-should end up being constructed by the students, with the teacher providing such help as is needed to produce it. In less fortunate cases it can lead to results like that reported by a young student quoted by Roth, Anderson, and Smith (1987): "1 already knew that plants need light to live. and now I know it again." Even in the more substantial examples that we have seen, there is a tendency to focus on the process rather than on the knowledge emerging from it, to abandon inquiry as soon as the principle has been found. and to keep the work Isolated from the body of knowledge already developed and accessible through books. All of this, we would suggest, reflects the folk view that recognizes only World I (physical action on physical objects) and World 2 (mental processes and content).

3. Learning through problem solving. Students are engaged collaboratively in some problematic task that might be anything from a complicated mathematics problem to doing an environmental impact study. A popular task these days is planning a space trip. In the course or planning the voyage, students must deal with a host of problems that space travel entails, and they are expected, as a result. to learn much about outer space, the solar system, human biology, rocket science, and general physics and mathematics. Such activity can be called knowledge building in only a marginal sense. The World 3 object that the students are objectively building is a plan for a space mission. Although they may make use of a variety of World 3 objects, such as scientific theories and accepted facts, the actual object they are building- the plan-is not an object of value In its own right. It is really a pedagogical vehicle for producing learning. Thus, the real point of the activity is the Incidental learning that results, rather than the building of knowledge.

4. Curiosity-driven Inquiry. Driven by their own curiosity, perhaps stimulated by challenging questions from the teacher, students gather information from reading, observation, or empirical research in an effort to satisfy their curiosity and answer the questions. Such activity counts as knowledge building insofar as it results in creating World 3 objects of value. Typically the main focus of attention is not World 3 but World 1 -the animals. electrical circuits, or whatever that are the actual objects of curiosity, plus the report or presentation that is the concrete product of the activity. World 3 objects do get produced-explanations, summaries of received knowledge, hypotheses, and the like-but they are seldom deliberately evaluated and improved in the course of Inquiry. Instead, it is left to the teacher to evaluate them, after the activity is ended. Efforts by teachers to get students to test hypotheses and to weigh evidence run Into difficulties that are usually attributed to the students' inadequate conceptions of theory, evidence, and scientific method

(Kuhn, 1989). However, it should be noted that such efforts to foster scientific thinking presuppose a World 3 focus that the structure of the inquiry activities may do little or nothing to support.

5. Theory improvement. Inquiry again begins with students' questions and puzzlements, but the focus is Immediately shifted to World 3 by having the students propose initial theories (we shall call them "theories" because that is what they are called In the classroom, although "conjectures" would be a more accurate term). The focus of inquiry and discussion then becomes improvement of these theories. Pursuit of information may go on much as in the preceding approach, but the information obtained is applied to theory improvement and its relevance is judged In that context. Scientific thinking does not grow out of efforts to test or substantiate certain beliefs but out of trying to show how a revised theory is an improvement over its predecessor (Scardamalia, Bereiter, Hewitt, and Webb, in press).

Although educational approaches within the range of these live levels are often treated as merely procedural variations on the same constructivist theme, they actually represent fundamental differences. There are differences in assumptions about students' motives and abilities, about how knowledge Is created, and about what learning needs to receive deliberate attention and what should be allowed to develop In Its own way through experience. These differences add up to radically divergent views of what schooling can or should be.

To gain a better perspective on knowledge building in schools, it Is necessary to go further into the implications of constructivism than is usually done. The first Implication is that learning-by-discovery, no matter how hands-on It may be, is not primarily a matter of discovering what the natural world has to reveal. It is a matter of discovering a World 3 object-a principle or theory-that makes sense of what is observed. These objects, of course, are not originally discovered; they are created by people trying to make sense of observations similar to those being made by the students. The second Implication is that, from the learner's point of view, there Is hardly any difference between creating a new theory or explanation and understanding one that has already been established. Popper made this point explicitly:

What I suggest Is that we can grasp a theory only by trying to reinvent It or to reconstruct It, and by trying out, with the help of our Imagination, all the consequences of the theory which seem to us to be interesting and important .... One could say that the process of understanding and the process of the actual production or discovery of . .[theories, etc.] are very much alike. Both are making and matching processes. (Popper and Eccles, 1977, p. 461)

Thus the learning side and the knowledge-building side of formal education merge in cases where the World 3 object that is being produced in the classroom is an interpretation, critique, or derivation of one or more culturally recognized World 3 objects. A powerful way of "getting to know" a World 3 object (and here we part company with the analogy to Newton's dog) Is to create another World 3 object based upon it. This is how science, scholarship, and to an Important degree the arts as well (Gombrich, 1959) progress. With an improved conception of knowledge and learning. it becomes easier to see how this kind of progressiveness can be brought into schools.

Learning and Knowledge Building in Schools: The Case of Elementary Mathematics

Research institutes and laboratories have knowledge as their end product. Hightech companies have knowledge as an intermediate product, something created on the way to producing the advanced goods or services that are their source of income. What is the status of the knowledge students produce in school? Is it merely an intermediate product. created in the service of learning? Essentially, yes, although the relation between the knowledge and the learning Is more complicated than a simple affirmative suggests.

No matter how much a school might be restructured to function like a research laboratory, society Is unlikely to accept World 3 knowledge as its end product. What society thinks It Is paying for and sending its offspring to school to obtain is World 2 learning: the personal knowledge and competence that students will carry away with them In their own minds. Whatever schools do in the way or promoting the building of World 3 knowledge will, accordingly, have to be justified on the basis or its contribution to World 2. To a large extent, this is true even of universities, where the production of new World 3 knowledge is part or their role. Periodic complaints that research is receiving too much emphasis are answered by claims that research improves the quality of education.

We would like to see a less restrictive attitude, with schools being recognized as having a dual role, the way universities do. This would not mean expecting scientific advances to come out of elementary schools. It would, however, mean seeing knowledge creation as a worthy societal activity that young students have some part In and seeing this activity as continuing and advancing to higher levels as students proceed to higher levels of schooling. Lacking this broader conception, the conventional view leads to an unfortunate devaluation of students' knowledge-building activities. They are reduced to learning activities. They are an alternative route to learning academic content-learning by discovery, as opposed to learning from direct Instruction-or they are a kind of academic role playing.

We are not arguing for the opposite here-for devaluing learning and making It a mere adjunct to knowledge building. Rather, our claim is that by failing to distinguish between learning and knowledge building, educators fail to do full justice to either. This can be shown most clearly In the case of elementary mathematics.

From a World 3 standpoint, mathematics Is the purest of cases. Formal mathematical knowledge consists of World 3 objects, but, unlike scientific knowledge, the objects do not refer to the physical world (Popper's World 1). Rather, they refer to other World 3 objects; in the case of arithmetic, they refer to numbers, which are also immaterial objects, constructions of the human intellect.

But school mathematics is not wholly concerned with World 3. Indeed, in its typical versions, school mathematics has scarcely anything to do with World 3- the world of mathematical ideas treated as improvable human constructions. Instead, It Is occupied with the Individual acquisition of skills (World 2), supplemented to a greater or lesser extent by activities involving World I objects, known as "manipulatives." Of course, manipulatives, such as base 10 blocks, are chosen because they embody important mathematical Ideas, but children are famously insensitive to the ideas and inclined to treat the physical objects as just physical objects (Resnick and Omanson, 1987). New approaches to school mathematics are emerging that do engage students in World 3 activity (Lampert, Rittenhouse, and Crumbaugh, this volume, chap. 3 1). Skills of some kind continue to be recognized as important, but the old skills or executing paper-and-pencil computations have declined in importance. Problem-solving skills have risen in prominence, but are these skills in the same sense. and are they leachable in anything like the way long division, for instance, is teachable? School mathematics is thus a challenging testbed for concepts of learning. In this discussion, we shall not presume to resolve any or the controversies surrounding school mathematics, but only to show how the enlarged set or concepts we have been proposing serves to draw useful distinctions.

Situated Quantitative Abilities

There is a growing and fascinating body of research carried out under the situated cognition label that examines the informal quantitative skills of street vendors, truck loaders, garment workers, and the like (reviewed in Lave and Wenger, 1991). The research shows that people with little or no formal education In mathematics learn how to carry out quite Impressive computations, using methods that are typically more efficient than those employing formal algorithms--methods that take advantage of peculiarities of the work and of the environment In which It is carried out. Not surprisingly, this research has created a pressure on schools to include more real-life kinds of quantitative activities In the mathematics curriculum. But is this a reasonable conclusion? A better grounded conclusion would be that schools should not bother with this sort of skill at all, because there Is no way they can roster it to good purpose. What the research reveals Is knowledge embedded in cultural practices. Each kind of practice has Its distinctive knowledge, which is acquired through extended and increasingly full participation in a particular community of practice. As an environment for fostering such skills, one would have to go some distance to design a worse one than the common school. To feel comfortable rejecting such obviously desirable learning objectives, however, educators would need a clear view of other kinds of learning to pursue as alternatives. For that, we need to see knowledge embedded In practice as one among several kinds of knowledge, others of which may be equally valuable and more suitable to the conditions of schooling.

Number Sense

We earlier defined number sense as "knowing one's way around" in numerical domains. It is a facility with numbers themselves, not with the quantitative aspects of particular environments or practical activities. Number sense, thus conceived, has been found to vary greatly among people. even if they are equally proficient in some situation-bound quantitative activity (Stigler and Baranes, 1988). It has also been found to vary greatly with socioeconomic status and to be highly predictive of progress in school mathematics (Griffin, Case, and Siegler, 1994). Most mathematicians, we may assume, have an abundance of number sense; yet one may look in vain through the guidelines and textbooks produced by mathematicians for anything that addresses the development or number sense in a knowing way. They do not seem to distinguish It from mathematical understanding. Within Popper's three-worlds framework. however. the two are quite distinct. Mathematical understanding relates to World 3-the world or mathematical principles. Number sense is pure World 2. It is a property or the Individual mind.

As we suggested earlier, the reason number sense is hard to get hold of conceptually is that it does not lend itself to the mind-as-container metaphor. If we are bound to this metaphor, then we look for mental objects to constitute number sense, and the obvious candidates are textbook principles, as replicated In the minds of students. But number sense cannot be found in any textbook. To be sure, any particular demonstration of number sense can be Justified by appealing to recognized mathematical principles. But a student could thoroughly grasp the principles and yet lack number sense.

Although number sense thus appears as something mysterious and elusive from the standpoint of folk psychology, It appears as something quite natural from the standpoint of a connectionist view of mind. It is a variety of pattern learning. It Is just what we would expect a connectionist network to acquire after extensive and varied Involvement with numerical relationships. Rightstart, the most successful and fully developed program for teaching number sense that we know of (Griffin, Case. and Siegler. 1994), provides just this kind of experience and can best be understood as tuning the whole cognitive system to numerical relationships rather than as Implanting any particular sorts of objects in the mind.

Knowledge Building in Mathematics

Drawing on the distinction made earlier between learning and knowledge building, we can confidently identify the development of number sense as learning. What, then. would constitute knowledge building in school mathematics? What are the World 3 objects students might create? The obvious answer is that knowledge building would mean producing the kinds of things mathematicians producetheorems, structures, algorithms. proofs, along with such subsidiary objects as explanations and justifications. Generally speaking, that is what goes on in the new constructivist approaches to school mathematics (Lampert, Rittenhouse. and Crumbaugh, this volume, chap. 31; Yackel, Cobb, Wood, Wheatley, and Merkel, 1990). Identifying this as knowledge building, a World 3 activity, has the virtue of setting It off as a distinctive kind of enterprise whose relationship to the rest of school mathematics needs to be considered. It leaves us free to ask a question that is difficult to put clearly in the language of folk psychology: What is learned as a result of this knowledge building? That question is important for determining to what extent knowledge building may replace other kinds of educational activity in mathematics and to what extent it supplements or reinforces them. Or is what is learned through knowledge building off on another dimension and of little relevance at all to traditional objectives? (See Ohlsson and Rees, 199 1, for an analysis and theoretical model that addresses questions of this kind.)

Popper's distinction between World I and World 3 also has a useful bearing on knowledge building in school mathematics. It helps us to understand why it is so much harder to motivate than knowledge building in empirical domains. We have found students to generate productive and challenging questions when invited to state what they wonder about in all sorts of areas of natural and social science (Scardamalia and Bereiter, 1992). But these same students (end to be dumfounded when asked what they wonder about in mathematics. They do not see that there is anything to question or wonder about with regard to mathematics.

One fifth-grade student tried to explain this lack of problematicity. "Say I have a problem about whether I have enough wood to make shelves in my closet. That's a problem for me, but It isn't a problem for anyone else. Somebody else might have a problem about gas for their motorbike, but what do I care about that?"

The unstated contrast here Is with problems in the empirical sciences, where students believe, with good reason, that a question that interests them is likely to interest others as well. Questions like "How do electromagnets work?" and "What causes people In the same family to look alike?" have generality, whereas questions about shelf length in a closet or fuel consumption In a motorbike do not. But, of course, questions about shelf length and fuel consumption are not mathematical questions. They are questions about particulars of a real or imagined World 1, which mathematics may serve as a tool In answering. But they are not very Interesting questions.

One approach to this motivational problem Is to introduce more Interesting World I situations. These can range from bringing real-life commerce Into the school (e.g., Richmond, 1973) to presenting complex fictional situations, as In the Jasper Woodbury videos (Cognition and Technology Group at Vanderbilt, 1992). Although this approach may motivate more extensive and meaningful uses of mathematics as a tool, It is not clear that it brings students any closer to the construction of World 3 mathematical objects. The other approach is to make numbers themselves sufficiently real to children that they begin to wonder about them, form conjectures, dispute, and Investigate. It amounts, virtually, to making numbers a part of World 1, the natural world, so that they are no longer just attributes of physical things but are things In themselves, capable of arousing puzzlement and wonder. This, It would seem, Is an essential part of the approach taken by Lampert and others, who engage students In the public construction of mathematical knowledge. We do not want to judge one approach against the other. only to point out that they are radically different, and that their being lumped together under the label of "constructivism" Is a testimony to the need for an enriched conceptual framework.

Procedural Learning

In discussing mathematics learning, we have so far slighted the kind or learning that often makes up virtually the entire mathematics program in elementary schools (and sometimes high schools as well). This is the learning of step-by-step procedures-commonly, procedures for adding, subtracting, multiplying, and dividing, first with small whole numbers. then larger numbers. then fractions and decimals; later, procedures for solving equations of various kinds. Our leaving this kind of learning until last should not be taken as a judgment on Its Importance. There were two reasons for putting it last. One was to make the point that there are very significant kinds of mathematics learning that are neglected in traditional school mathematics, and that this neglect is at least in part traceable to a prevailing folk theory of knowledge and learning. The other reason is that, when it comes to procedural learning of the kind commonly pursued in elementary mathematics, learning theories grounded In folk theory seem to be on solid ground.

In the prevailing usage within cognitive science, procedural learning and skill learning are synonymous. What Is learned consists of rules in the mind-"production systems," In the most fully developed theory of skill learning (Anderson, 198 3: Anderson, 1987; Anderson. Conrad, and Corbett, 1989). The difference between learning to execute a multidigit subtraction algorithm and acquiring number sense, according to this view. is entirely a matter of the size and complexity of the rule system to be learned. Because we see them as Importantly different, we would prefer to limit the term "procedural learning" to learning of the first kind, where explicit procedures are involved, and to use the term "skill learning" more broadly to refer to acquiring the vast range of human competencies-everything from walking to creative writing-that go beyond what can be reduced to explicit procedures.

With cognitive theories grounded in the mind-as-container metaphor, learning multidigit subtraction Is relatively easy to explain and number sense is difficult. We already know the rules for doing multidigit subtraction right, and so it only remains to account for the various erroneous rules that students may concoct (Brown and Burton. 1978). Number sense, by contrast, is baffling because there seem to be countless context-sensitive rules. From a connectionist standpoint, the opposite Is the case. Number sense Is just the finely tuned adaptation to environmental conditions that all sorts of learning are expected to exhibit. Learning how to execute a fixed procedure under the guidance of explicit rules, however, raises difficult problems about how a cognitive system can in some fashion consult rules stored within It (Beer, 1991). It may be remarked that this seems to be the right order of difficulty. Any organism credited with an intelligence exhibits adaptive capabilities similar to number sense, whereas the consulting of explicit rules may be unique to human intelligence.

Accordingly, folk psychology, with its modern derivatives, is at its best when dealing with procedural learning that involves rules that are explicitly stated or demonstrated. Even if connectionist learning theory is eventually able to account adequately for such learning, systematized folk theory is likely to remain more practical for dealing with It instructionally. When a student is doing the wrong thing. but in a seemingly deliberate and orderly fashion then a practical first step in helping the student is to try to cast the student's behavior in the form of rules (some of which the student may be able to report explicitly) and then consider what might have prompted such rules, what their adaptive value might be in the instructional context, and what might be required to get the student to abandon those rules in favor of more satisfactory ones. Although from a connectionist standpoint such rule-based models are fictions and will usually provide only an approximate fit to the student's actual behavior-behavior that often deviates even from rules that the student explicitly claims to be following (Nisbett and Wilson. 1977)-they can be useful fictions in dealing with problems of procedural learning. They can also be useful for dealing with problems of learning bodies of knowledge already explictly formulated, as in textbook physics. They can be dangerous fictions, however, when applied to more complex and intuitive kinds of skill and understanding, such as number sense.

Conclusion

Central to Western folk theory of mind is the metaphor of mind as container. Until recently, this metaphor has shaped every theory in which mind figures as a concept. Learning, accordingly. has been conceived or as the Introduction and modification of objects in people's minds, with theories differing as to the processes involved. We have argued that the container metaphor Is too limiting, and that education needs an expanded set or concepts if it is to do full justice to the various roles of knowledge in modern lire. There are Important kinds or learning that cannot be reduced to objects In the mental container. To do justice to them, we can make use or the new connectionism, which offers us the conception of a mind that can act knowledgeably and logically without Itself having content. Sociocultural theories offer us a conception or knowledge that inheres in cultural practices and artifacts and that cannot be reduced to either the contents or the capabilities of individual minds. Popper's three-worlds schema offers a unifying framework In which we may conceive of a connectionist mind (World 2) acting knowledgeably and logically with respect to knowledge objects that have a kind or autonomous existence outside both individual minds and cultural practices. It Is these objects, constituting Popper's World 3. that are created, evaluated, revised, and otherwise operated upon In the advancement of science and, more generally, in what is coming to be called "knowledge work."

Within this expanded conceptual framework, a distinction between knowledge building and learning becomes possible. Knowledge building is activity directed outward toward World 3; learning is activity directed Inward toward changes in World 2, one's own mental abilities and dispositions. Both are valuable, and it now becomes possible to consider questions such as the following: How much of schooling should be devoted to knowledge building and how much to learning? What is learned from particular knowledge-building activity? How can each support the other? In the preceding section we elaborated such questions as they arise with respect to school mathematics, but they are questions of importance in any area. Yet they are questions that tend to get lost in the kinds of conceptual soup dished up under labels such as "constructivist learning" or "higher-order thinking and learning."

It is not only educators who could benefit from a more elaborated conception of knowledge and learning. As Olson and Bruner propose (this volume. chap. 2), students' own conceptions of knowledge and learning are important. If there is a more profitable way to conceive of knowledge and learning than the way offered by folk theory, students ought to be let in on the secret. If today's students are to be making careers out of working with and adding value to knowledge, they too need a conception or knowledge and learning that does not confuse knowledge objects (World 3) with mental states of knowing (World 2). This is not an unrealistic goal. even at the elementary school level. Students in one class heavily committed to knowledge building (Scardamalia, Bereiter, Hewitt, and Webb, in press) were asked individually how they knew when they had learned. The most articulate response was given by a fifth-grade girl. but other responses were along the same lines. She said:

> I think that I can tell if I've learned something when I'm able to form substantial theories that seem to fit In with the information that I've already got; so it's not necessarily that I have everything, that I have all the information. but that I'm able to piece things in that make sense and then to form theories on the questions that would all fit together.

We doubt if Karl Popper would have put it much better.

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