Fostering constructivist-oriented mathematical beliefs through knowledge-building

Abstract: This case study investigated the impact of engaging teacher-education students in knowledge building on their mathematical beliefs. In particular, an idea-centered instructional design was introduced to facilitate knowledge-building processes. Data analyses focused on (a) idea improvement process as documented in a Knowledge Forum database, and (b) a mathematical beliefs survey. Results showed that idea-centered knowledge building was able to help the participants develop more constructivist-oriented mathematical beliefs.

Research shows that beliefs are closely related to learning experiences (Pajares, 1992; Richardson, Anders, Tidwell, & Lloyd, 1991; Schommer, 1994; Wilson, 1990). If students' learning experiences are related to more didactic instructional approaches, it is more likely that they will develop more absolutist-oriented beliefs. As commonly observed in conventional mathematics classrooms, such belief tends to be fostered through encouraging students to rely on textbooks or teachers as authoritative knowledge sources (Cooney, Shealy, & Arvold, 1998; Green, 1971; Schoenfeld, 1989; Szydlik, Szydlik, & Benson, 2003). In contrast, when learners are prompted to learn through more discovery-guided instructional approaches, they are more likely to develop constructivist-oriented beliefs. Similarly, teacher-education students' beliefs can also closely relate to their learning-to-teach (teaching practices) experiences. To help teacher-education students cultivate more productive mathematical beliefs, the present study employed 'knowledge-building' in a mathematics teaching course.

Knowledge building, also known as "deep constructivism" (Scardamalia, 2002, p. 4), is defined as a social process focused on sustained production and improvement of ideas of value to a community (Scardamalia & Bereiter, 2006). Unlike the conventional view of education that highlights learning through acquiring and accumulating well-established knowledge (Paavola, Lipponen, & Hakkarainen, 2004; Sfard, 1998), knowledge-building employs ideas as building blocks for deeper knowledge around a specific topic. The importance of valuing ideas as basic units of thought or objects of inquiry can be manifested by means of Popper's (1972) 3-World epistemic conceptualization. Popper refers to World-1 as an objective, natural/physical/material world, World-2 as a subjective psychological world constructed within the human mind, and World-3 as a conceptual world constituted mainly by ideas (e.g., theories, models). He argues that ideas are the creative results of human beings (such as engineers, scientists, researchers, artists, and the like) and that all forms of human knowledge are related to the creation of ideas in a human community (Scardamalia, 2002). Bereiter (2002) further argues that ideas are conceptual objects which, once produced in a public domain, can possess a social life of their own and can be continually tinkered with, modified, and improved.

Unfortunately, conventional views of education tend to focus on learning through knowledge acquisition and accumulation (e.g., understanding World-1 by changing students' mind in World-2), but not working creatively with ideas (e.g., transforming students into knowledge workers in World-3) (Bereiter, 1994; Paavola, Lipponen, & Hakkarainen, 2004; Sfard, 1998). Similarly, teacher-education students are unaccustomed to the ways of assuming the role of theory-builder or knowledge-worker as teaching professionals. Instead, they are often encouraged to pursue exemplary teaching practices after some model teachers. If teacher-education students do not know how to work innovatively with ideas as knowledge-workers, it is questionable that they will be able to guide school pupils to develop the kind of innovative competencies essential in knowledge-based societies (Hong, 2011; Zhang, Hong, Scardamalia, Teo, & Morley, 2011). Thus, in addition to learning about content-based knowledge and exemplary teaching practices, it is equally important to provide teacher-education students with opportunities to learn to work with ideas for building knowledge.

Previous research on in-service teachers who have been practicing knowledge-building pedagogy for years suggests that such practice may stimulate epistemological growth among these teachers (Chai, Wong, & Bopry, 2009; Chai & Tan, 2009; Zhang, Hong, Scardamalia, Teo, & Morley, 2011). Building on this line of research, it is posited that engaging teacher-education students in collaborative knowledge-building should also have effects on their views about the subject matter they are to teach and their teaching capacity. Yet such assumption remains to be examined, especially in the domain of mathematics.

Method

Study Design, Participants and Instructional Context

This case study attempts to gather rich data embedded in a course context. The participants were nine teaching-education students (four females and five males) and their age ranged from 19 to 23 years ($M = 21$; $SD=1.59$). They were planning to become middle-school mathematics teachers in Taiwan after graduation, so took a university-level course entitled Middle School Mathematics Teaching. The course was offered by the university's Center of Teacher Education; the university is ranked as one of the top 10 universities in the nation. The course served a practical purpose as it represented a final course requirement before students graduate and begin their student-teaching internship. Before taking this course, students needed to complete most theory-based courses—for instance, instructional theories and adolescent psychology—as prerequisites. The main instructional goal of this course was to foster adaptive practices and dispositions in mathematics teaching. Major instructional and research activities throughout the academic year were organized as follows:

(1) A pre-post belief survey was conducted at the beginning and end of the study to measure participants' mathematical belief changes. This was done using open-ended questions concerning the nature of mathematics and that of ideal mathematics teaching and learning (see below for details).

(2) A tutorial workshop about how to use Knowledge Forum (KF) was administered in the first two weeks of the school year. Students were introduced to some basic functions of KF, for instance, creating a note in a KF "view" (i.e., an online problem-solving space) or building on a note.

(3) For the remaining time in the academic year, the participants were engaged in knowledge-building. In particular, an idea-centered instructional approach, proposed by Hong and Sullivan (2009), was employed to foster sustained knowledge-building. This instructional approach was developed based on a review and has yet to be empirically tested; the present study was the first to examine this approach. Under this approach, ideas are improved in two dimensions: quality and quantity. From a social perspective, the quality of ideas is a function of how knowledge workers (epistemic agents) collaboratively work with ideas, and the quantity of ideas is a function of how ideas (conceptual objects) are shared and/or exchanged in a community. Building on Popper's (1972) evolutionary epistemology, ideas may be substantially refined in quality by means of constructive elaboration, or significantly enriched in quantity by means of continued diversification. One thing to note is that one-sidedly focusing on either idea elaboration or idea diversification may lead knowledge-building activities into a less productive path. For example, research shows that keeping ideas as one's intellectual property without sharing with, or obtaining new perspectives from, other members can impede knowledge creation in a company or a community (Chubin 1976; Granovetter 1983). On the other hand, merely sharing ideas or information with others in a community (e.g., social networking) does not warrant the transformation of ideas into deeper understanding (Kling & Rosenberg 1986). A more balanced and productive trajectory to sustained idea-improvement relies on the transformation of ideas both in quality and quantity through an emerging process of self-organization that is enabled by simple rules (e.g., idea elaboration and exchange) to gradually form a complex network of ideas (Prehofer & Bettstetter, 2005). Based on this instructional design approach, participants were explicitly guided to engage in the following three idea-improvement activities:

(a) Idea generation: Participants were guided to generate and work on their initial teaching ideas; accordingly, they worked on lesson plans, set instructive goals, prepared learning materials and worksheets, etc. Then, based on their ideas, they performed their teaching practices in class, with the other classmates serving as the audience and critical reviewers.

(b) Idea exchange and diversification: This activity facilitated idea diversity and sharing from multiple perspectives. To generate ideas for feedback, participants were guided to ask questions such as: "If you were to teach this same lesson, how would you do differently to improve the teaching practices?"; "What is your main idea?"; "Why is it useful?"; "How is it going to improve teaching?", etc. They then posted their ideas in the form of a note in the Knowledge Forum.

(c) Idea elaboration and reflection: Next, the student who completed his or her teaching practice would go online to review and summarize all ideas and feedback provided by peers, reflect on previous teaching practice, and try to improve and prepare for the next teaching practice. In addition, the participants were required to write reflection notes at the end of each practice and a reflection paper at the end of the course.

In summary, the activities were designed to support sustained knowledge building. It is important to note, however, that the order of the three activities was not at all fixed, as the process of idea generation, exchange, and elaboration could occur at any time during the knowledge building process.

Knowledge Forum—an Online Knowledge-Building Environment

In addition to tutorial workshop activity and teaching practices, which were held physically in class, all other activities (e.g., contribution of teaching ideas, peer-feedback, peer-assessment, and self-reflection, etc.) were held in the Knowledge Forum (KF). KF is an online platform that runs on a live database. It allows users to simultaneously create and post their ideas in the form of a note on a database, read others' postings, watch videos, reply to others' notes, search and retrieve records, and organize notes into more complex knowledge representation. KF runs in both a text and a graphics mode. In the graphics mode, it shows linkages of postings as a way to represent the interconnectivity and dialogical nature of knowledge. Within the KF, users are guided to work as a community by making explicit their problem of interest, producing initial teaching ideas, sharing and connecting ideas, synthesizing ideas, and deepening collective understanding of the problems at issue. Specifically for this study, a key problem of interest in the course was concerned with improving teaching practices and attaining deeper understanding of the nature of mathematics, mathematics teaching, and mathematics learning.

Data Source and Analysis

The main datasets came from (a) participants' teaching ideas posted online as notes, and (b) a pre-post belief survey. First, online data were recorded in a Knowledge Forum database. Using ideas (defined as distinct suggestions for improving teaching practices) as units of analysis, content analysis was performed to examine patterns of peer-feedback and self-reflection for the improvement of students' teaching practices (Strauss &

Corbin, 1990). What emerged from open coding was three areas of improvement (including learning content, instructional method, and personal performance) and two courses of idea improvement (i.e., ideas generated for improving either teacher-centered or student-centered practices). For the purpose of analysis, the three idea improvement stages were divided into: stage 1 (between the first and second practices), stage 2 (between the second and third practices), and stage 3 (between the third and fourth practices). A repeated-measures ANOVA was computed to test if there were any significant changes among the three stages of idea improvement. To compute inter-rater reliability, two coders independently categorized each idea. A Kappa coefficient was calculated to be .77.

Second, the pre-post belief survey was developed based on Handal's (2003) conceptualization of mathematics beliefs in three aspects: views of the nature of mathematics, views of mathematics teaching, and views of mathematics learning (see also Ernest, 1991). A previous study by Tsai (1998) investigating students' epistemological beliefs in natural sciences used a belief survey with eight open-ended questions. This study adopted this same survey, with minor text revision (e.g., changing the word 'science' to 'mathematics'). The eight questions are as follows: (1) What is mathematics? (2) What does doing mathematics mean to you? (3) What is an ideal way to teach mathematics? (4) What are some key factors for successful mathematics teaching? (5) What makes an ideal mathematics teacher? (6) What is an ideal way to learn mathematics? (7) What are some key factors for successful mathematics learning? (8) What makes an ideal mathematics learning environment? Of the items, questions 1 and 2 concern the nature of mathematics; questions 3 to 5 concern the nature of mathematics teaching; and questions 6 to 8 concern the nature of mathematics learning. Content analysis was employed (Strauss & Corbin, 1990) using a pre-determined coding scheme developed based on the above conceptualization of mathematics beliefs (Handal, 2003) (see Table 1). Wilcoxon signed rank tests were conducted to measure if there were any pre-post belief changes. Two coders independently performed the coding process. The inter-coder kappa was calculated to be 0.95.

Table 1. Coding scheme of mathematical beliefs

Category	Sub-category	Example
Absolutist-oriented beliefs: Regarding mathematics as a set of tools, consisting of formulas, theorems and theories. Students need to master the use of tools in order to achieve teaching objectives (Ernest, 1988).	**Mathematics**: is a science (or group of related sciences) dealing with number, quantity and measure (Risteski et al., 2008).	- Mathematics is geometry, algebra, statistics, probability, number, quantity, etc.—a combination of different mathematical knowledge and [tools]. (S1). - Math is a science about calculating numbers. (S04)
	Mathematics teaching: is to train students' thinking ability.	- I think Mathematics is a subject that trains and exercises our brain. (S2). - The best way to teach a math course is to lecture, using the simplest and most straightforward way to explain concepts in order to help students understand them, as complex mathematics builds upon simple mathematical facts and concepts. (S1).
	Mathematics learning: is to acquire basic mathematics concepts and procedures and to practice again and again.	- Practice makes perfect. (S3) - The more you think, try, and practice math quizzes/problems the better you can solve similar quizzes or problems and understand the concepts and facts that are required to solve these problems. (S5)
Constructivist-oriented beliefs: Mathematics is a course of dynamic exploration and creative invention. The course includes making mistakes and sustained revision and correction. Mathematics does not necessarily represent absolute truth or eternal knowledge, but can be validated or falsified by continual exploration and improvement (Ernest,	**Mathematics**: is a science of exploring patterns, orders, and relations (Franklin, 1994).	- Doing mathematics is to seek for patterns or principles by means of given conditions, using symbols and numbers to predict, estimate, or conjecture possible outcomes. (S9) - Math is a way to find patterns and orders in life, through the use of symbols and numbers and that of logical thinking…math provides a means to knowing the world, exploring rules in complex affairs, and reducing errors. (S4)
	Mathematics teaching: is to help students develop their own way of mathematics learning, and	- It is (a) to make students like math and be interested in it; (b) to want to explore a math problem in depth and discuss with others about it; (c) to be willing to collaborate with others and try

1988).	to guide them to explore and solve problems, through discussion and collaboration.	various means collectively to solve problems. (S9). - I think teaching is not to lecture myself, but is to provide opportunities for students to explore math in a natural way, to frequently interact with students and to motivate students to think about problems, to allow students to try and learn from their own mistakes, by giving them enough time to think and discuss among themselves; one-way talking will be unlikely to motivate students to learn. (S8).
	Mathematics learning: is to develop one's own way of understanding through mathematical problem-solving.	- It is to establish one's own learning style by learning how to learn math and by working and discussing with others; by accumulating such experiences, one will not be limited to one's habitual ways of thinking and will be able to think from multiple perspectives, and be able to come up with even better solutions to the same math problem. (S6). - Learning is to explore and identify a more systemic way for one's own math learning and to gradually develop more effective learning processes. (S02).

Results

Idea improvement

Content analysis on students' notes was performed to illustrate how the participants produce and improve ideas. The results revealed that a total of 516 ideas were contributed in the KF throughout the school year. These ideas mainly came from two sources: peer feedback and self-reflection. A non-parametric Wilcoxon signed-ranks test showed a significant difference between the two sources in terms of the percentage of idea contribution, with more ideas coming from peer feedback (M=71.7%, SD=11.4%) than self-reflection (M=28.3%, SD=11.4%; z=-2.67, p<.01). Further, in terms of areas of idea improvement, it was found that ideas mainly contributed to improving teaching practices in three areas: learning content, instructional method, and personal performance. A non-parametric Friedman test showed a significant difference among the three areas, with significantly more ideas being contributed to improve instructional method (M=24.22, SD=7.10) than in the two other areas (i.e., personal performance, M=18.67, SD=4.95; and learning content, M=14.44, SD=3.32; χ^2 =12.06, p<.01). Moreover, in terms of the course of idea improvement, using participants as units of analysis and repeated-measures ANOVA, it was found (see Figure 1) that there was a progressive decrease in terms of the percentage of the number of participants' ideas being contributed to improving more teacher-centered teaching practices. The percentage in the three improvement stages was 66.3% (SD=15.0%), 57.0% (SD=15.5%), and 46.3% (SD=15.6%) respectively (Wilks' lambda=.403, F=5.19, p<.05, η2=.60). In contrast, there was a progressive increase in terms of the percentage of the number of participants' ideas being contributed to improving more student-centered teaching practices. The percentage in the three improvement stages was 33.7% (SD=15.0%), 43.0% (SD=15.5%), and 53.8% (SD=15.5%) respectively (Wilks' lambda=.413, F=4.98, p<.05, η2=.59). As a case example, to illustrate how the participants progressively move away from more teacher-centered to more student-centered idea improvement, shown below is the way in which a participant (S4) collaborated with peers and worked on ideas to improve her teaching practices in the areas of learning content, instructional method, and personal performance. This case was selected as the participant's teaching is highly teacher-centric and she mainly relied on lectures in her first teaching practice, as compared with other participants.

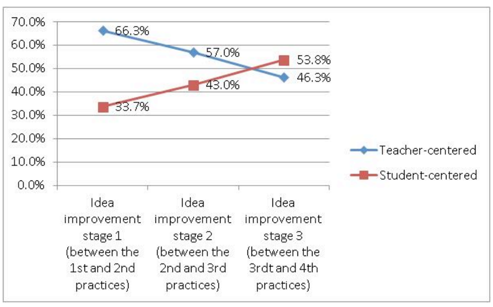

Figure 1. Two courses of idea-improvement in teaching practices (teacher-centered vs. student-centered)

First, in terms of learning content, S6 suggested to S4: "From a student point of view, I think the examples you used for teaching should be simpler because students were not familiar with this new concept [i.e., congruent triangles] that you were introducing." S6 thought S4 did not put herself in students' positions, finding that the test items S4 prepared in the worksheet were too difficult for the students; she did not take students' prior understanding into her teaching consideration. In response, S4 reflected: "The test items I used were selected from national competency tests with which I was familiar. I was not conscious that they were too difficult. I will think again what test items to include next time." So peer feedback promoted S4 to reflect on how to select test items that are more appropriate from the viewpoint of students. Second, regarding instructional method, S2 told S4 that "there was too much lecture and because you were mainly lecturing, your teaching heavily relied on the use of the textbook. You may try to integrate some visual aids or illustrations into your teaching, so as to better engage students." In this case, S2 shared her personal ideas about how to motivate students to learn. In response, S4 elaborated, "using visual aids is a good idea. When I was preparing this lesson, I thought that the textbook already has figures in it, so lecture alone is good enough. I agree that using figures can be helpful for increasing learning interest." In this case, peer feedback helped S4 to shift her teaching focus to student learning and motivation, having previously focused too much attention inward. Finally, in terms of personal performance, after observing S4's teaching practices, S6 suggested: "I think you should raise your voice and maintain comfortable eye contact and posture with the students at all times". In response, S4 wrote, "I was very nervous during my whole teaching. Maybe it was because I am not a very confident person. I guess my nervousness is also because I am afraid of dealing with unplanned events that might occur during teaching. This is definitely an area that I want to improve in my next practice." Clearly, peer feedback also prompted S4 to be aware of her highly teacher-centric teaching style.

Changes in mathematical beliefs

Content analysis was performed on the data derived from the pre-and-post belief surveys to answer the third research question of whether the instructional activities affected teacher-education students' mathematical beliefs. Overall, regarding general epistemological views in mathematics, as Table 2 shows, the non-parametric Wilcoxon signed rank tests showed that there was significant decrease in ratings from pre-survey to post-survey in terms of absolutist-oriented views ($z=-2.25$, $p<.05$); in contrast, it was found that there was significant increase in ratings from pre-survey to post-survey in terms of constructivist-oriented views ($z=-2.67$, $p<.01$).

Further analyses were conducted to look into the three specific aspects of the epistemological views (beliefs in the nature of mathematics, beliefs in mathematics teaching, and beliefs in mathematics learning). First, regarding absolutist-oriented views, a significant pre-post change was found only in participants' beliefs in mathematics teaching ($z=-2.23$, $p<.05$). There was no significant pre-post change in participants' beliefs regarding the nature of mathematics and belief in mathematics learning. Possibly, this was due to the small sample size. Alternatively, it may be because, to a certain degree, students still believed that memorization of mathematical facts is needed as a base for higher levels of mathematics learning. On the other hand, it was found that all three aspects of the constructivist-oriented views showed significant pre-post changes ($z=-2.39$, $p<.05$, in terms of beliefs in the nature of mathematics; $z=-1.98$, $p<.05$, in terms of beliefs in mathematics teaching; and $z=-2.53$, $p<.05$, in terms of beliefs in mathematics learning).

Table 2. Participants' mathematical beliefs

Mathematical views	Pre-survey		Post-survey		*z-value*
	M	SD	M	SD	
Absolutist-oriented beliefs	9.89	4.40	4.56	2.79	-2.25*
- Mathematics: is a science (or group of related sciences) dealing with number, quantity and measure	3.67	1.87	2.11	2.37	-1.13
- Mathematics teaching: is to train students' thinking ability	4.11	2.42	1.89	1.36	-2.23*
- Mathematics learning: is to acquire basic mathematics concepts and procedures and to practice again and again.	2.11	2.2	0.56	0.73	-1.7
Constructivist-oriented beliefs	0.89	1.05	10.22	6.63	-2.67**
- Mathematics: is a science of exploring patterns, orders, and relations	0.00	0.00	2.56	2.07	-2.39*
- Mathematics teaching: is to help students develop their own way of mathematics learning, and to guide them to explore and solve problems, through discussion and collaboration	0.67	0.87	3.67	4.42	-1.98*
- Mathematics learning: is to develop one's own way of understanding through mathematical problem-solving	0.22	0.44	4.00	2.06	-2.53*

*<.05 **<.01

Conclusions and Implications

It is thought that helping pre-service teachers develop the necessary skills and attitude for lifelong learning is of great consequence to the teaching profession (Bereiter, 2002). To address this challenge, the present study focused on an instructional shift—from learning-to-teach by following a lesson 'script' (Adams & Engelmann, 1996; Engelmann, 1980; Sawyer, 2004; Slavin & Madden, 2001), to learning-to-teach by working innovatively with 'ideas' (Bereiter, 2002). While scripted teaching practices can help teacher-education students acquire greater abilities in routine teaching performance with high efficiency, such mode of teaching might also lead practitioners into a comfort zone and develop a mental habit that is inclined to seek a strong sense of security (White, 2009). Instead, guiding teacher-education students to work innovatively with ideas for teaching practice is more likely to help them move beyond thinking about routines to try out new teaching strategies and adjust what they are doing, developing progressively more effective and personalized teaching practices (Hammerness et al., 2005).

Knowledge-building theory has been developed over the past 20 years (Scardamalia & Bereiter, 2010) and has been recognized as a foundational approach to learning sciences (Sawyer, 2006). As 'deep constructivism' (Scardamalia, 2002, p. 4), knowledge-building attempts to guide classroom activities away from proceduralized tasks to innovative knowledge work (Zhang, Hong, Scardamalia, Teo, & Morley, 2011). In a special issue of the *Canadian Journal of Learning and Technology on knowledge-building* (Jacobsen, 2010), a set of studies ranging from the elementary-school classroom setting to campus classrooms provided convincing examples of what students can achieve in knowledge-building classrooms in the advancement of knowledge. In the present study, the findings further suggested that engaging teacher-education students in sustained knowledge-building in a teacher-education course could also help the teacher-education students develop beliefs that view teaching as creative and improvable practices (contrasted with beliefs that view teaching as ritualized activities). In conclusion, this study shows that the proposed idea-centered instructional design was viable for guiding teacher-education students to develop more adaptive teaching beliefs. Admittedly, there are limitations that must be recognized in this study. There is a need for greater consideration regarding generalizability from a single class of nine teacher-education students; further research is needed in more diverse class contexts.

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