A Knowledge Building Discourse Analysis of Proportional Reasoning in Grade 1

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Abstract: Proportional reasoning can be defined as the consideration of number in relative terms as opposed to absolute terms. This reconceptualization requires for the learner a shift from additive to multiplicative reasoning—a shift that has been shown to be challenging for both children and even many adults. In this study we use the discourse analysis tool Knowledge Building Discourse eXplorer (KBDeX) to analyze the growth and extent of the use of proportional language in a grade one classroom as students engage in material over the course of four specially designed discourse-based lessons in proportional reasoning. The design of the first three lessons (the intervention) was based on psychological and educational research and employed a context integrating both continuous and discrete representations of proportions. The fourth lesson served as an assessment and involved the students solving proportional reasoning problems featuring discrete quantity of the types typically found in math textbooks for students in older grades. Content analysis was conducted on recorded discourse, focusing on students' multiplicative reasoning and knowledge building behaviours. KBDeX was then used to track frequencies and connectedness of students' multiplicative reasoning as well as knowledge building behaviours across lessons. The social network of students involved using multiplicative reasoning and knowledge building behaviours was also analyzed. Findings reveal that knowledge building behaviours and multiplicative reasoning (multiplicative comparisons, multiplicative operations, and grouping language) increased between lesson 1 and lesson 4 and that the majority of the students were able to solve the final assessment tasks presented in the fourth lesson. Finally, we present a discussion on the potential of knowledge building discourse for student learning in math.

Keywords: discourse, knowledge building, math, proportional reasoning, young children

Introduction: A Background on Proportional Reasoning

Proportional reasoning can be difficult to define. As Van DeWalle (2006) states, "it is not something that you either can or cannot do but is developed over time through reasoning. It is the ability to think about and compare multiplicative relationships between quantities." Lanius and Williams (2003) refer to proportional reasoning as a mathematical way of thinking defined by the ability to recognize proportional situations and to use multiple approaches for solving problems involving proportionality. Fernandez, Llinares, Van Dooren, De Bock, and Verschaffel (2009) make the distinction between within-variable and between-variable relationships in proportional contexts. 'Within' relationships compare quantities of the same nature, while 'between' relationships compare quantities of different nature. The authors assert that competence in proportional reasoning involves not only the ability to solve proportional problems, but also the possession of a deep understanding of the multiplicative relationships between quantities, including the comprehension and use of both within and between relationships. The Ontario Ministry of Education (MOE; 2012) refers to proportional reasoning as simply the consideration of number in relative rather than absolute terms. All these definitions refer to a very important component of math development that encompasses many aspects of the math curriculum (i.e. equivalent fractions, converting units of measurement, probability, multiplication and division, money amounts, rates of speed etc.). Proportional reasoning is also known to be conceptually very difficult for middle school students (Lamon, 2007; Mitchelmore, White, & McMaster, 2007) and even adults (Lamon, 2007). Exceptional teaching practices are required to ensure a deep understanding of proportional reasoning.

Proportional reasoning has been thought of as a topic for older children. The Ontario curriculum does not mention proportional relationships until grade four (MOE, 2005). Similarly, Piaget and Inhelder (1975; Inhelder & Piaget, 1958) have documented that children are not capable of proportional reasoning until about the age of 11.Van Dooren, De Bock, and Verschaffel (2010) found that students in the younger grades almost always incorrectly apply additive strategies to proportional problems. In contrast, other studies have shown that children as young as five (Sophian, 2000; Sophian & Wood, 1997), six (Schlottman, 2001), and seven-years old (Goswami, 1989) can partake in proportional thinking in different contexts. In particular it was discovered that young children can be successful at proportional reasoning when the problem context involves reasoning with continuous, rather than discrete representations (Mix, Huttenlocher, and Levine 2002; Jeong, Levine, & Huttenlocher, 2007). Boyer, Levine, and Huttenlocher (2008) found that children go astray on proportional reasoning tasks when they are asked to match two proportions given in numerical (i.e, discrete) quantities. It has been suggested that students might benefit from proportional reasoning tasks that integrate continuous with discrete contexts or representations. With this kind of integration in mind, Moss and Case (1999) developed a successful experimental lesson sequence for the teaching and learning of rational number that grounded students' initial understandings of proportion in a linear measurement context of continuous quantity using the numerical language of percents (discrete representation). The analyses revealed that this learning context involving the integration of discrete and continuous quantity played an important part in the development of the students' understanding of rational number and proportion (Moss, 2005). Since proportional reasoning takes time to develop and is not a result of natural growth (Koeller-Clark & Lesh, 2003), it should be introduced to children even younger than the curriculum suggests.

The present case study uses a sequence of lessons designed by Moss, Comay, Stephenson, and Halewood (in preparation) with grade one students exploring the concepts of intuitive (or continuous) and numerical (discrete) proportionality. In this lesson sequence, as we describe in detail in upcoming sections, proportional reasoning tasks were introduced in a fantasy context that used snakes of various lengths (continuous quantity) and numbers of magic pellets (discrete quantity).

Discourse and Mathematics Learning

Current reforms in mathematics education advocate the establishment of mathematical learning communities in classrooms to support students to engage in productive mathematical discourse. Indeed the importance of mathematical discourse—"math talk" —has been central in reform mathematics education literature in both North America and internationally for over 25 years (NCTM, 2000; National Research Council, 2001). Underlying this shift towards discourse in the teaching and learning of mathematics is the idea that mathematics is primarily about reasoning and not memorization; it is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Central to the move towards discourse is the notion that mathematics should be taught in a way that mirrors the nature of the discipline (Lampert, 1990). In this model of mathematics learning, the classroom functions as a community where thinking, talking, agreeing, and disagreeing is encouraged in order to discover important mathematical concepts (Bruce, 2007). Indeed, numerous research studies reveal that students learn mathematics best when they are given opportunities to explain their mathematical reasoning using the language of mathematics (Kazemi & Stipek, 2001). As Martino and Maher (1999) point out, the opportunities to engage in discourse not only increase student's abilities to problem solve but also increase students' engagement in the subject matter.

However, while research points to the potential benefits of math talk to promote student learning, many studies reveal the problems of discourse classrooms. Indeed, the model of math learning in which students' ideas serve as the basis for class discussion has proven to be challenging for both teachers and students.

From a teacher's perspective, this form of discourse can be difficult to implement. Kilpatrick, Swafford, and Findell (2001) have suggested that managing discourse is one of the most complex tasks of teaching. Sherin (2002) aptly likens the teacher's role in trying to promote sustained, productive, and meaningful math discourse to a "balancing act" wherein teachers operate within a tension of supporting a student-centered process of mathematical discourse and, simultaneously, facilitating discussions of significant mathematical content.

A discourse-based math learning environment also involves many challenges for students. For example, Baxter, Woodward, and Olson (2001) raise the concern that math talk may not suit all students. Many students do not know how to explain their mathematical ideas and are uncomfortable with expressing their understandings (Empson, 2003; Walshaw & Anthony, 2008).

However, the greatest challenge is the change in the role of student from passive recipient of math knowledge to an active and responsible member of a learning community (epistemic agents who understand their role as contributors to knowledge). Cazden (2001) points out that each student becomes a significant part of the learning environment, and that in a math discourse classroom, teachers depend on students' individual contributions for advancing learning in the class (p. 131).

Exploring the Use of Knowledge Building in Mathematics

The importance and challenges of discourse in mathematical learning are clear. In the present study we experimented with a knowledge building approach to discourse to support grade one students to use multiplicative reasoning for proportions. We speculated that a knowledge building approach might maximize the quantity and quality of discussion amongst the students and thus support them in gaining a strong foundational understanding of this very difficult topic.

Knowledge building (KB) pedagogy focuses on collaborative learning experiences where students can openly negotiate their ideas with each other, in which the goal is to improve the community's understanding as a whole (Chiarotto, 2011). As Scardamalia and Bereiter (2003) put it, "knowledge building results in the creation or modification of public knowledge—knowledge that lives 'in the world' and is available to be worked on and used by other people." In a knowledge building classroom, a student proposes her ideas to the whole class and the responsibility to improve those ideas rests on all students in the class acting as a community. In other words, students doing knowledge building take collective responsibility for continually advancing their community knowledge (Scardamalia, 2002). Thus, shared discourse, face-to-face or online, is central to knowledge building.

In the present study the teacher engaged the students in KB discourse using a circular seating configuration known as a knowledge building (KB) circle (Chiarotto, 2011). The advantage of a KB circle is that it can encourage attentive listening and communication, diminish hierarchy, and foster inclusive respect. A KB circle is not organized according to scripted procedures or rituals (Zhang, Hong, Scardamalia, Teo, & Morley, 2011). Rather, it operates according to a set of twelve knowledge building principles (Scardamalia, 2002), such as community knowledge and collective responsibility, and progress in an emergent manner towards knowledge advancement.

Method

The present study consists of three intervention sessions on proportional reasoning in the form of knowledge building lessons. The lessons were then followed by an extended discussion where the students were given the opportunity to apply their knowledge to different contexts which assessed their proportional reasoning skills.

Participants

Eleven grade one students act as participants of the knowledge building circle which was moderated by their teacher. The school culture is one in which knowledge building pedagogy is a central focus and the students have had previous experience using Knowledge Forum to engage in the learning of science topics. These students were sorted by their teacher as the higher achieving half of their class. A few researchers were also present to film and take field notes on the sessions.

Materials and Apparatus

The materials for the lessons consisted of rectangular construction paper strips representing a long snake, *Longy* (14 x 2cm), a short snake, *Shorty* (7 x 2cm), and *Baby* snake (3.5 x 2cm). Circular counters were also used to represent magic pellets required for the snakes to perform magic tricks.

Multiple video cameras were used to record the lessons and to capture the discourse between students in a clear manner. A total of two hours of videos were then transcribed using a personal computer and Microsoft Excel. Analysis was conducted using KBDeX software.

Procedure: The Lessons

Each of the four lessons was half an hour in length (described in detail below) and used different participation structures. Lesson 1 took place in the form of a knowledge building circle. Lesson 2 was mostly comprised of small group discussions. Lesson 3 was a presentation sequence of student-designed challenges. Lesson 4 (like lesson 1) was a knowledge building circle for extension problems.

Lesson 1. To begin the first lesson, the students were presented with two rectangular construction paper strips, one of which was half the length of the other. The teacher told the students that these two strips were actually snakes named *Longy* and *Shorty* and asked the students to comment on the relationship of the length of one snake to the other. Once the students offered that the short snake was half the length of the tall snake, they were given their own pairs of rectangular strips and asked to prove this relationship. The goal of this part of the lesson is to have each child experience the 1:2 relationship in the context of continuous quantity—length.

Next, they were introduced to the discrete assignment of numerical values in the form of magic pellets the snakes required in order to perform magical feats. They were told and shown using the circular counters that *Longy* needed exactly four pellets and *Shorty* needed exactly two pellets to dance on the CN Tower, and then were asked why they thought these were the numbers required. Next, missing-value problems were introduced in the following order.

"If Shorty needs five pellets to sing opera, how many might Longy need?"

"If Longy needs eight magic pellets to play the guitar, how many will Shorty need?"

"What if we wanted Longy and Shorty to build a tower at structures, out of blocks. Can someone tell me how many pellets Shorty might need and Longy might need to do that?

The lesson ended with the introduction of "*Baby* snake" a 3.5 x 2 cm rectangle, similar to the opening of the lesson, students were asked to prove the 1:4 relationship of the length of *Baby* to *Longy*.

Lesson 2. The lesson took place two school days after the first lesson. A few new researchers were present so the students were asked to explain to the new visitors what they discovered the other day about the lengths of the snakes. Next, students were asked to find a partner and to design their own missing-value challenge for the class using *Baby* and *Longy* (discrete, numerical values for the 1:4 relationship). The lesson concluded with a re-gathering; one group got to share their work with the time that permitted.

Lesson 3. The third lesson took place three school days after the second lesson. Students presented their invented missing-value challenges to their classmates.

Lesson 4. The last lesson served as an informal assessment offering students the opportunity to extend their knowledge and to attempt to solve proportional reasoning problems in purely discrete, numerical contexts. The lesson began with a brief review of the preceding lessons; children were asked to explain the proportional relations of the lengths amongst the three snakes (continuous relations) and numerical proportions of the pellets (discrete relations), and how the lengths of the snakes related to the numbers of magic pellets required for the snakes to perform magic acts. Next the students were presented with a pair of proportional reasoning problems in written form with accompanying illustrations. The two problems were typical of textbook-style proportional reasoning problems and were as follows:

"If 5 candy canes cost 10 cents. How many candy canes can you buy for 30 cents?"
"If you need 5 pellets to feed 2 fish. How many pellets do you need to feed 8 fish?"

Analysis of Data

Coding. The transcript of the discourse was coded in two different ways. The first was for different types of knowledge building behaviours and the second was for different types of multiplicative reasoning.

Knowledge Building Behaviour Codes.

The following three codes were based directly on knowledge building theory and were used to analyze the different ways that the students contributed to the discourse.

Build-on (bdo): A discourse unit was coded as a *build-on* when the student adds to the conversation, whether it's a new idea or another strategy (E.g. after a child reports that one snake is longer than the other one, another child adds "one's half the size of the other.").

Extension (extn): A discourse unit was represented as an *extension* when a student offers a novel idea to a problem either because it is the first response or because it is a contribution that hasn't been brought up before (E.g. a student offers the representation of fractions, "the fraction would be, um here's the centre line. The fraction would be 1 out of 2 because two of these blue pieces would make up to one whole.").

Collaboration (clln): A discourse unit is coded as *collaboration* when a student explicitly references a peer's idea (E.g. "I'm building onto Nichola's"), make clear reference to a peer's idea (jumping in while students are offering ideas/ responding; E.g. "no that would be half"), or making *we* statements (ex. "*we* figured it out" or "*we* think")

Multiplicative Reasoning Codes.

Multiplicative Action (ma): when students refer to or actually fold or cut with scissors or hand gestures. E.g. *"So I folded this; and then I opened it back up and then I put this on to see if it was half, and it was."*

Multiplicative Comparison (mc): when students refer to at least two multiplicative situations in the same explanation. This could include either within-variable or between-variable reasoning. E.g. *"I think that's it's always going to be that Shorty's always gonna have half less than Longy because he's half shorter, so it's always half lower."*

Multiplicative Operation (mo): when students refer to a specific operation such as multiply by, divide by or when they compute using multiplication or division. E.g. *"But you can't divide 100 by 40."*

Fraction Language (fl): when students use fraction language in explanation. E.g. *"The fraction would be 1 out of 2 because two of these blue pieces would make up to one whole."*

Count By (cb): when students either explicitly or implicitly suggest counting by strategy. E.g. "*'cause if you count by twos - two, four, six- by counting by twos and then you would put four."*

Grouping Language (gl): when students either explicitly refer to grouping for multiplication or it is inferred. E.g. *"you use two twos"* meaning "you use two (groups of) two."

Halving Language (hl): when students refer to half as a part of a whole, E.g. "*this snake is a whole and this snake is a half"* or half as a multiplicative operation of halving. E.g. *"half of four is two."*

Off Task (ot): when students lose focus with the discussion; may include irrelevant features.

KBDeX.

The discourse was analyzed using a relatively new social network analysis tool called Knowledge Building Discourse eXplorer (KBDeX; Oshima, Oshima, & Matsuzawa, 2012). Given a transcript of discourse, the tool analyzes social networks of learners, based on co-occurrences of a pre-selected list of terms, which can be domain vocabulary, phrases, or content analysis codes. Given a discourse transcript and a list of terms, KBDeX goes through each discourse unit to check co-occurrence of terms. If two terms co-occur in the current unit, a link will be drawn between them; similarly, if two discourse units share a same term, a link will be drawn between them too. At the same time, by attributing each discourse unit to its contributor (e.g., student), KBDeX further infers links between students based on accumulated linkages of discourse units contributed by them. In the end, KBDeX will create a social network of students, a network of discourse units, and a network of terms, all based on co-occurrence of terms. By visually and interactively exploring these three types of networks, researchers can investigate relationships among students from very specific angles. For instance, by interpreting the student network, KBDeX allows one to see which students are integrating terms or phrases similarly to other students and which ones are more isolated in terms of conversation content. By checking the network of tracked terms, KBDeX can display the connectedness of codes in a transcript and provide significant insights about the discourse content. By comparing these networks across different discourse phases, researchers could further inspect changes of discourse across time represented by structure of terms and students. In the present study, by tracking multiplicative reasoning and knowledge building behaviour coding, KBDeX was used to answer our primary research question of whether there will be an increase or movement to multiplicative language across the lessons as the tasks become more discrete in nature.

Research Questions.

The present case study attempts to address the following specific research questions.

Knowledge Building Behaviours.

1) Did the students collaborate, build onto, and extend each other's ideas? Is there an increase in these behaviours between lesson 1and lesson 4?
2) Did all the students participate in working on the proportion challenges? Were there any off-task behaviours? Were they using a collective use of language?

Multiplicative Reasoning.

3) Was there an increase in the number of multiplicative reasoning codes between lesson 1 and lesson 4?
4) Did the students make use of both within and between-variable reasoning, in solving the challenges in lesson 4?

Potential Benefit of KB for Multiplicative Reasoning.

5) Is there a co-occurrence of the KB codes and multiplicative reasoning codes that is greater in lesson 4 than lesson 1?

Results and Discussion

In our analyses we compared lesson 1 to lesson 4. As we described earlier in preceding sections of the paper, both lesson 1 and 4 used the same knowledge building circle format. Both lessons two and three were different. One involved small group activity and the other involved presentation of student-created problems. Therefore for the purpose of these analyses we compared multiplicative reasoning and knowledge building behaviours of lesson 1 and lesson 4.

Knowledge Building Behaviours

The defined *build-on, extension,* and *collaboration* codes were detected abundantly throughout the transcripts. As table 1 indicates, the proportion of each of the coded discourse units to the total number of discourse units were much larger in lesson 4 than lesson 1, suggesting that the children adapted to the KB set-up and made use of these approaches in advancing their knowledge as a group. When one child built onto another child's idea, further ideas manifested. See Table 1 for these values.

Table 1

Proportion of Discourse Units Coded with KB Behaviours

Knowledge Building Behaviour	**Lesson 1**	**Lesson 4**
Build-on (*bdo*)	19%	47%
Extension (*extn*)	19%	31%
Collaboration (*clln*)	5%	14%

All children were immersed in the discourse and were actively participating. See Figure 1 for a visual representation from KBDeX reflecting the extent to which students were knowledge building and multiplicatively reasoning similarly to their peers. The thickness of the lines indicates the extent to which each student displayed knowledge building behaviours and multiplicative reasoning similarly to other students.

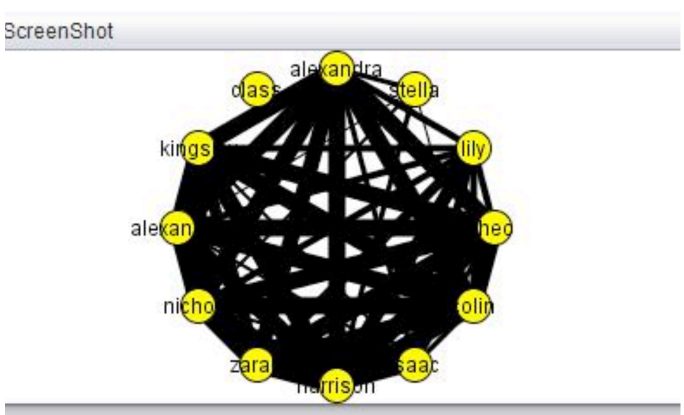

*Figure 1.*Student network based on co-occurring knowledge building behaviours and multiplicative reasoning. The teacher and researchers were taken out of the analysis.

A frequency count of only four discourse units, by two students reflected *off task* behaviours demonstrating a high level of engagement as well.

Multiplicative Reasoning

Absence of Additive Reasoning.

Initially we also coded for additive reasoning, as the literature states young children begin their proportional reasoning using additive strategies (Van Dooren et al., 2010). However, inconsistent with the literature, there was less additive reasoning happening initially (two instances in total). This can be accounted for by the knowledge building structure of the lessons which have been revised many times as a part of a Japanese lesson study (Moss, Comay, Stephenson, & Halewood, in preparation). The absence of additive references could be because the lesson and the questions were structured in such a way to divert thinking away from additive reasoning. For instance, the teacher presented *Longy* with four pellets and *Shorty* with two pellets and asked the students why they think these are just the right amounts. The KB circle allowed for discussion to be steered towards more accurate strategy use and misconceptions were quickly addressed by one's peers through verbal reasoning. Since this was a classroom where knowledge building is a central focus, the students have gained a disposition to partake and contribute to discussions.

Growth in Multiplicative Comparison and Operations and Grouping Language.

Findings also indicate that proportions of discourse units involving *multiplicative comparisons, multiplicative operations*, and *grouping language* were all greater in lesson 4 than lesson 1 indicating more multiplicative reasoning displayed in the discourse by the time the intervention was complete. *Counting by* language increased slightly, *multiplicative action* and *fraction language* both decreased which can be observed as being due to the nature of the lessons. Lesson 1 involved discussion on showing that *Shorty* was half of *Longy* and thus involved folding strategies and a discussion on how he is 1 out of 2 in fraction form. *Halving language* also decreased which can be expected since the handover to discrete/ numerical proportionality would have encouraged less emphasis on "half" or "halving" and more on multiplicative and divisional operations. These values can be viewed in Table 2. Overall, the students became more multiplicative in their reasoning. Particularly impressive was the way students began to use the language of multiplicative operations by the 4th lesson because in this class the students had no formal instruction in either multiplication or division. The use of *multiplicative comparison* language- that students refer to two separate multiplicative relations in a single explanation – revealed a sophistication of reasoning pointing to a deep understanding of the proportional relations in the problems.

Table 2

Proportion of Discourse Units Coded with Multiplicative Reasoning

Multiplicative Reasoning Type	**Lesson 1**	**Lesson 4**
Multiplicative Action *(ma)*	13%	0%
Multiplicative Comparison (*co)*	6%	31%
Multiplicative Operation (*mo)*	3%	28%
Fraction Language (*fl)*	14%	6%
Counting By (*cb)*	1%	8%
Grouping Language *(gl)*	3%	11%
Halving Language (*hl)*	14%	8%

Within and Between-Variable Reasoning.

As stated in the introduction, Fernandez et al. (2009) assert that a deep understanding of proportional reasoning is evident in the use of both within and between-variable relationships. The difference between them can be illustrated with an example; if one can purchase 2lbs of cherries for $8, and one wishes to purchase 5lbs, to determine the cost one could use within or between reasoning; to determine the cost using 'within-variable' reasoning relates lbs to lbs (2lbs to the 5lbs desired (2:5)) and then one can apply this ratio to the cost. If one engages in between-variable reasoning, one compares lbs to dollars (2lbs to the $8 cost (1:4)).

In this study, we analyzed the use of within and between-variable reasoning in lesson 4. Our analysis revealed that the group came up with three different ways of explaining within-variable reasoning and two different ways of explaining between-variable reasoning when solving the candy cane problem. An example of a within-variable explanation is as follows, "I know that three…ten times three is thirty and five times three is 15 so it must be 15." Here the student compares cost to cost (10 times three is 30) and number of candy canes to number of candy canes (5 times three is 15). An example of a between-variable explanation is as follows, "um it's like two ways… one is half because that (points to five candy canes) is half of ten." Interestingly, one child used another strategy outside of within and between reasoning, "I realized that um if you ... it's two, each candy cane is two cents and half of 30 is 15." This child used the strategy of unitizing – determining how much one candy cane costs and applying this unit price to the new situation. The more difficult fish and pellet question yielded two different within-variable descriptions and a between-variable description. A student who was reasoning within-variables stated, "I looked and saw that there's two and if you divided this (8 fish) by two it's four and I did four times five is 20." This student used the 2:8 or 1:4 relationship to reason. Between-variable reasoning was stated as follows, "so we saw that two fish need five pellets so two fish make five then two more need 10, two more 15, and two more 20 (2:5 relationship)." One child even explained why he thought another child came to a different answer, "so 2 is 5 so you go five, 10, 15, 20 so you just count by fives except you go like that so you don't do 5, 10, 15, 20, 25, 30, 35, 40, 'cause I think that's what Theo and Harrison did 'cause they didn't look at it's 2 fish equal 5. These students were making great use of discourse and verbalizing their thinking, but one can also see that they have come to very advanced understandings of discrete proportions for children of their age, having used so many different strategies.

Potential Benefit of KB for Multiplicative Reasoning

Although it cannot be said for sure, it is suggested that the knowledge building context had great influence on this outcome of such multiplicative thinking. The network of coding—knowledge building and multiplicative reasoning codes—displays a noteworthy enhancement between lesson 1 and lesson 4. Despite the different participation structure in lesson 2 (small group discussions), we noticed students were making use of both *build-ons* and were using more accurate multiplicative reasoning; although the actual analysis compares lesson 1 and 4 due to the similar KB circle structure of the lessons, the network is also displayed from lesson 2 which shows an appropriate middle-ground between lesson 1 and 4. It must also be noted that not all the small group discussions were captured in lesson 2. Figure 2 displays the connections among these codes from lessons 1, 2, and 4; thicker lines indicate more frequent co-occurrences within the same discourse units.

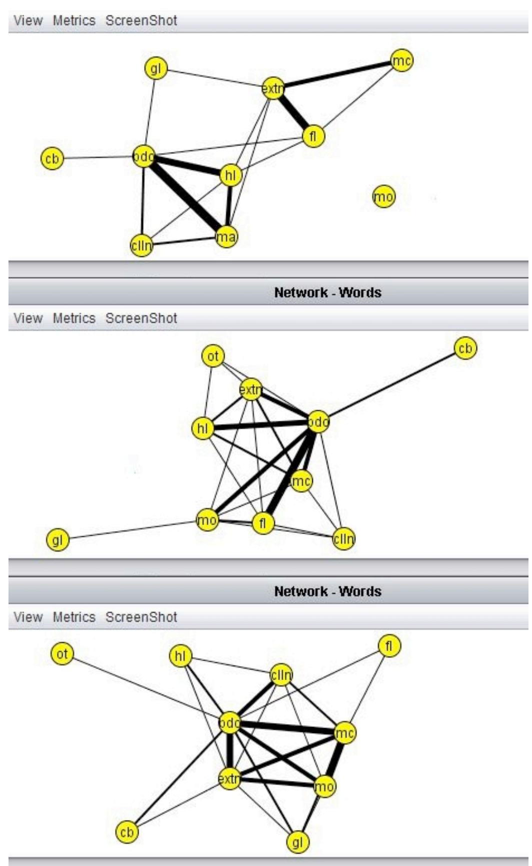

*Figure 2.*This analysis looked at the co-occurrence of knowledge building behaviours and multiplicative reasoning in lessons 1, 2 and 4. *Multiplicative comparisons* (*mc*) and *multiplicative operations* (*mo*) are particularly associated with the KB behaviours in lesson 4.

In lesson 1 the salient connections are between *extensions, fraction language*, and *multiplicative comparisons* and also between *build-ons, halving language*, and *multiplicative actions*. These are noteworthy since they involve both KB behaviour and multiplicative reasoning, however there is a lack of cohesiveness and overall connectedness. Notice that multiplicative operations are not connected to any other code in lesson one; this means students were not using ‘multiplied by’ or ‘divided by’ language at this point while using other reasoning strategies and knowledge building behaviours.

In lesson 2, *build-ons* took a central role in the discourse as one can see in Figure 2. Thick lines indicate strong connections between *build-ons* and *extensions*, *halving language, multiplicative operations, fraction language,* and *multiplicative comparisons*. The students have begun using extensions that are also classified as *build-ons* (whereas before an *extension* may be merely presenting the first idea to the teacher’s question, now students are advancing the groups knowledge by presenting new ideas) while also incorporating multiplicative operations. An example from the discourse that represents a *build-on, extension, multiplicative comparison*, as well as *halving language* is as follows, “because if *Shorty* is half of *Longy*, then he'll always have less pe- ... half of the number of pellets that *Longy* has.” His new idea is a generalization that had not been suggested at this point, but he is also building onto the conversation. Multiplicative operations became integrated; for example, with these quotes, “but you can't divide 100 by 40, because you can't divide 10 by 4 and so you can't divide …” and “you're dividing it by 4 because baby is... baby...you can have four *Babies* to equal *Longy*.” Here *multiplicative operations* are being used in conjunction with *build-ons, extensions*, *multiplicative comparisons*, and *fraction language*.

Finally, lesson 4 displays a noteworthy web of connections between *build-ons, extensions, collaborations, multiplicative comparisons,* and *multiplicative operations*. Here is an example, “I did it on the back of the sheet I did 10 cents equals five 20 cents is two tens 20, and then five plus ten and then 30 another five 15.” This student is presenting a new idea, building on the conversation, making a *multiplicative comparison*, and is using *multiplicative operations*. The students came to make use of the knowledge building behaviours to foster their multiplicative reasoning. The thicker lines between the mentioned five codes which indicate that the knowledge building strategies and the multiplicative reasoning are associated which supports the hypothesis that the knowledge building culture of the lessons may be responsible for the advanced, sophisticated outcomes.

The lessons began with an intuitive, continuous depiction of proportional reasoning that young children have been shown capable of understanding. Since an intuitive model was explored first, the connections to discrete representations were made more easily through verbal reasoning. To date research shows that young children have great difficulty understanding proportionality when it is presented in the form of numerical applications. The present study demonstrates that an understanding of proportion is possible for children as young as six-years-old when approached appropriately. Furthermore, as outlined in the literature, mathematical discourse is an emphasized and important feature of math learning. However, there are many challenges reported for both students and teachers in math talk classrooms. We speculate that the extent and quality of math discourse would be greatly enhanced if we used a knowledge building framework for the introduction to proportional reasoning. Our results reveal that a knowledge building discourse structure appeared to support students in making gains in their understanding of proportion. The students in the present study made use of knowledge building discourse which resulted in the sharing and refinement of their knowledge (Chiarotto, 2011). The analysis of the transcripts reveals that the knowledge building discourse structure encouraged participation from all students. Based on the large number of *extensions* noted in the transcripts, epistemic responsibility was held by the students to share their ideas and build on them. In summary, the knowledge building environment, coupled with the intuitive-to-numerical learning context are likely to have been factors leading to these gains in children's understanding.

Conclusion

It is well known in the research literature that proportional reasoning can be challenging to students of all ages. In addition, it is known that while a math discourse classroom is highly desirable to enhance students' mathematics learning, this kind of discourse class is challenging to achieve. The advanced use of multiplicative reasoning and students' abilities to solve extension problems shown by these grade one students reveal that in suitable circumstances, even young students can reason proportionally. In addition, the degree of interaction and student input into the class discussions that were discovered in this study support the benefits of a knowledge building framework. Finally, the association between the multiplicative reasoning and the knowledge building behaviour suggests that they do tend to occur together and perhaps knowledge building behaviours foster multiplicative reasoning. However, there were limitations to this study. Since we did not have a control group and did not pre-test the children, it is difficult to make claims about the growth of their understanding. We also cannot know whether different types of school settings would yield the same results. Further research might explore the relationship between knowledge building and mathematics learning.

Acknowledgements

I'd like to thank my supervisor and colleague of this project, Dr. Joan Moss for her continued support and guidance. She has taught me lot about conference preparation, the research process and about the literature on proportional reasoning and the importance of discourse in mathematics. I'd also like to extend thanks to the members of Joan's research lab who helped with the data collection.

Thanks also to JICS grade one teacher, Zoe Donoahue for teaching the lessons and for allowing us to collect data in her classroom. The research could not have been possible without her and her students.

A special thanks also to Jun Oshima and Yoshiaki Matsuzawa who guided us through analyses using KBDeX, as well as Dr. Marlene Scardamalia and Dr. Carl Bereiter for their professional input that contributed to the project.

Relevant Conference Themes

Intellectual Engagement

Sustained Work with Ideas

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